# Structure-Based Water-Filling Algorithm in Multipath MIMO Channels

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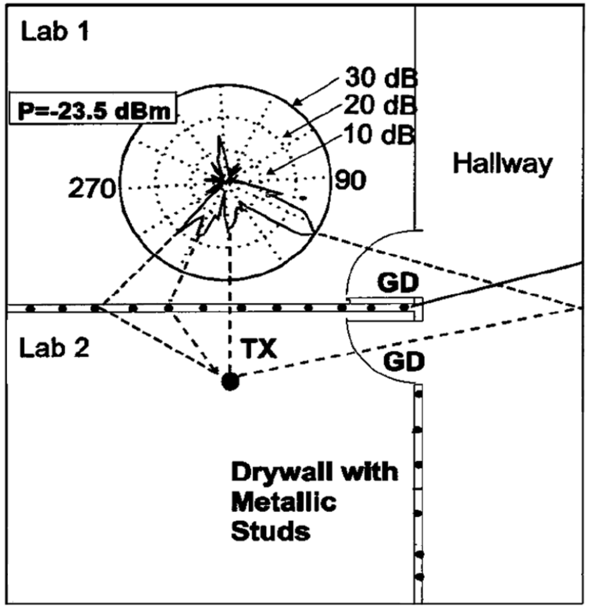
## Motivation

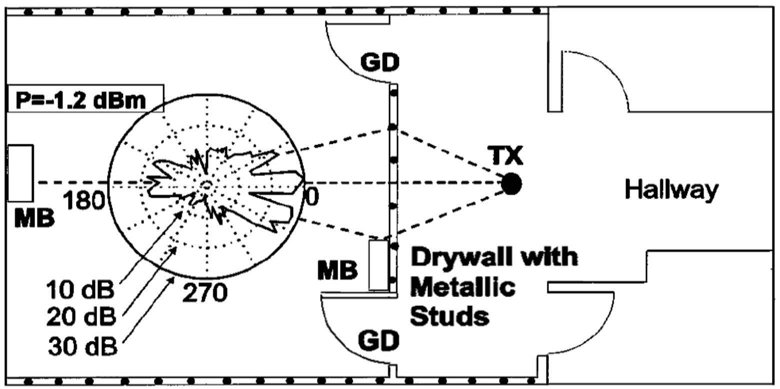
- With channel state information (CSI) at the transmitter, Raleigh in 1998 proposed an optimum space-time eigen-beamforming scheme in terms of system capacity.
  - However, eigenbeams are usually fast time-varying.
  - High complexity on MIMO channel tracking and large amounts of CSI feedback.
- Current and upcoming indoor wireless applications:
  - 5GHz for wireless LAN 802.11a
  - even up to 60GHz in the future wireless standards

- Wireless measurements:
  - The path directions-of-departure/arrival (DODs/DOAs) of these dominant paths have a strong dependence on the path delays.
  - Unlike the wireless channels in either urban or rural outdoor environments, the indoor wireless channel is characterized by reflection rather than diffuse scattering.
  - The variations of the dominant path delays are little and the variations of the dominant path DODs/DOAs are confined within limited angle spreads.

- Indoor wireless measurements




(a)




(b)

- Wireless multipath channel structure:
  - Angle selectivity: due to the path DODs/DOAs of multipaths.
  - Frequency selectivity: due to the delay spread of multipaths.
- Goal:
  - To propose a new space-time transmit scheme to exploit the wireless multipath channel structure.
  - A structure-based water-filling algorithm is employed in the new transmit scheme.

## Structured Channel Model

The receive signals $\mathbf{r}$ at the receive antenna array for all the $L$ dominant paths

$$\mathbf{r} = \sum_{l=1}^{L} \beta_l \Big( \underbrace{(\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l})\mathbf{a}_{\mathrm{T}}^{T}(\theta_{\mathrm{T},l}))}_{M_{\mathrm{R}}\times M_{\mathrm{T}}} \otimes \underbrace{\mathbf{G}_l}_{(N+V-1)\times N} \Big) \cdot \underbrace{\mathbf{f}}_{(M_{\mathrm{T}}N)\times 1} \cdot \underbrace{s}_{1\times 1} + \mathbf{n}, \qquad (1)$$

- $\mathbf{a}_R(\theta_{R,l})$ and $\mathbf{a}_T(\theta_{T,l})$ are respectively the receive and transmit array steering vector of the $l^{th}$ path.
- $\beta_l$ is the complex path fading amplitude of the $l^{th}$ path.
- $\mathbf{G}_l$ is constructed with the sampled pulse-shaping function for the $l^{th}$ path $g(nT - \tau_l)$; $\tau_l$ is the propagation delay of the $l^{th}$ path.
- $\mathbf{f} = [f_1^1\ f_2^1 \cdots f_N^1\ f_1^2 \cdots f_N^{M_{\mathrm{T}}}]^T$ is the transmit space-time weight vector with $s$ being the input data.
- $\mathbf{n}$ is the additive Gaussian noise.

## Space-Time Transmit Scheme

A. Receive Energy Maximization

- From the transmitter's viewpoint, it intuitively intends to deliver as much energy to the receiver as possible.
- Consider the following constrained problem

$$\mathbf{f}_{\rm opt} = \ \text{Arg} \ \underset{\mathbf{f}}{\text{Max}} \ \mathcal{E}\{ \ \| \sum_{l=1}^{L} \beta_l \Big( (\mathbf{a}_{\rm R}(\theta_{{\rm R},l}) \mathbf{a}_{\rm T}^T(\theta_{{\rm T},l})) \otimes \mathbf{G}_l \Big) \mathbf{f} s \ \|^2 \ \}$$

$$\text{subject to } \operatorname{tr}(\sigma_s^2 \mathbf{f}\,\mathbf{f}^*) = E_s,$$

where the total transmit power $\operatorname{tr}(\sigma_s^2 \mathbf{f}\,\mathbf{f}^*)$ is constrained to $E_s$.

- The best space-time eigenbeam, $\mathbf{f}_{\text{opt}}$, is solved as

$$\sum_i \sum_j \rho_{ij}(\mathbf{Q}_j^* \mathbf{Q}_i)\mathbf{f}_{\text{opt}} = \lambda_{\max} \cdot \mathbf{f}_{\text{opt}}, \qquad (2)$$

  where $\rho_{ij}$ denotes the $(i,j)^{th}$ entry of the covariance matrix of $\boldsymbol{\beta}$, $\mathbf{R}_{\boldsymbol{\beta}}$; $\mathbf{Q}_l \triangleq (\mathbf{a}_{\text{R}}(\theta_{\text{R},l})\mathbf{a}_{\text{T}}(\theta_{\text{T},l})^T) \otimes \mathbf{G}_l$; and $\lambda_{\max}$ is the largest eigenvalue of $\sum_i \sum_j \rho_{ij}\ (\mathbf{Q}_j^* \mathbf{Q}_i)$.

- The best subchannel for the transmitter to transfer signal energy to the receiver is the dominant eigenvector of $\sum_i \sum_j \rho_{ij}\ (\mathbf{Q}_j^* \mathbf{Q}_i)$.

- Note that the influence of the fast time-varying path fading amplitudes $\{\beta_l\}$ is averaged out and excluded from $\sum_i \sum_j \rho_{ij}$ $(\mathbf{Q}_j^* \mathbf{Q}_i)$.

B. Structure-Based Water-Filling

- In the $M_\mathrm{T}N$-dimensional signal space seen at the transmitter, if we transmit all the data through a single-dimensional eigenbeam,
  - either the MIMO system will have a very limited throughput,
  - or serious signal collision will be unavoidable.
- Parallel transmission is obviously an indispensable feature in the design of a MIMO system in order to maximize the data throughput.

- From the transmitter's viewpoint again, the *orthogonal* eigenvectors of $\sum_i \sum_j \rho_{ij}(\mathbf{Q}_j^* \mathbf{Q}_i)$ represent parallel subchannels to allocate energy for parallel data transmission.

- A heuristic water-filling solution is applied for these subchannels:

  – Apply eigen-decomposition,

$$\sum_i \sum_j \rho_{ij}(\mathbf{Q}_j^* \mathbf{Q}_i) = \mathbf{U} \ \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{M_{\mathrm{T}}N}) \mathbf{U}^*. \qquad (3)$$

  – Let $\mathbf{u}_i$ be the $i^{th}$ column of $\mathbf{U}$ and denote the partial-CSI-based eigenbeams seen at the transmitter.

– Impose the water-filling solution based on $\{\lambda_i\}$ and then obtain the space-time weight vectors as

$$\mathbf{f}_i = \sqrt{p_i}\mathbf{u}_i \quad (4)$$

$$\text{with } p_i = (\nu - \frac{\sigma_n^2}{\lambda_i})^+, \; i = 1, \cdots, M_{\mathrm{T}}N, \quad \sum_i p_i = \frac{E_s}{\sigma_s^2},$$

where $p_i$ is the energy assigned to the $i^{th}$ eigenmode; $\nu$ is a constant chosen so that the total transmit energy is equal to $E_s$; $\sigma_n^2$ is the noise variance; and $(\cdot)^+$ is an operator that produces zero when its argument is negative.

C. Performance Measure

- With $M_{\mathrm{T}}N$ orthogonal transmit carriers $\{\mathbf{f}_i\}$ available at the transmitter, $M_{\mathrm{T}}N$ data symbols $\{s_i\}$ can be transmitted simultaneously. Therefore, $\mathbf{f} \cdot s$ becomes

$$\mathbf{s} = \sum_i \mathbf{f}_i \cdot s_i, \tag{5}$$

where the variance of $s_i$ is $\sigma_s^2$ for all $i$.

- Assume that the input data $\{s_i\}$ are Gaussian and independent. The transmit signal vector, $\mathbf{s}$, is then Gaussian with its covariance matrix being

$$\mathbf{R}_{\mathbf{s}} = \sigma_s^2 \mathbf{U} \cdot \mathrm{diag}(p_1, \cdots, p_{M_{\mathrm{T}}N}) \cdot \mathbf{U}^*. \tag{6}$$

- With perfect CSI, $\mathbf{H} = \sum_{l=1}^{L} \beta_l \big( (\mathbf{a}_{\mathrm{R}}(\theta_{\mathrm{R},l}) \mathbf{a}_{\mathrm{T}}(\theta_{\mathrm{T},l})^T) \otimes \mathbf{G}_l \big)$, known at the receiver, the mutual information between the channel input signal and the channel output signal is given by

$$C = \log_2 \det(\mathbf{H}^* \mathbf{R}_{\mathbf{n}}^{-1} \mathbf{H} \cdot \sigma_s^2 \mathbf{U} \mathrm{diag}(p_1, \cdots, p_{M_{\mathrm{T}}N}) \mathbf{U}^* + \mathbf{I}_{M_{\mathrm{T}}N}), \quad (7)$$

where $\mathbf{R}_{\mathbf{n}}$ is the space-time noise covariance matrix.

- Outage capacity $C_{\mathrm{out}}(p)$,

$$\mathrm{Prob}\{\ C < C_{\mathrm{out}}(p)\ \} = p, \quad (8)$$

as the measure of the achievable performance is evaluated with Monte Carlo methods. We focus on the 10% outage capacity, i.e., $C_{\mathrm{out}}(0.1)$.

## Numerical Experiments

- Both sides of the transceiver are equipped with five-element ULAs, the antenna elements in both of which are spaced half a wavelength apart.
- We randomly generate one hundred channels with different combinations of path DODs, path DOAs, and path delays, each with 10,000 channel path fading amplitude realizations.
- Three algorithms are tested
  - The case with perfect CSI is the system employing the optimum eigen-beamforming solution.
  - The case with partial CSI represents the proposed system.
  - The case with no CSI denotes a system where the transmit signal is Gaussian distributed with covariance $\frac{E_s}{M_{\mathrm{T}}N}\mathbf{I}$.

- The input data block sizes considered in Figs. 2 and 3 are $N = 2$ and $N = 3$, respectively.
- The SNRs, defined as $\frac{E_s}{N_0}$, in both Figs. 2 and 3 are 5 dB, 10 dB, and 15 dB.
- We observe that, for over 90% of the channels, the capacities of the proposed transmit scheme at SNR=15 dB are 0.8 and 1.6 bits, respectively, less than the optimum transmit scheme.
- Compared with the transmit scheme without CSI at the transmitter, by exploiting the partial CSI, the proposed transmit scheme has 3.1 bits and 4.4 bits capacity advantage at SNR=15 dB, respectively, in Figs. 2 and 3.

- It is also observed that, at a higher SNR, there is a larger 10%-outage-capacity gap between the proposed transmit scheme and the one without CSI at the transmitter.
- Another interesting observation is that, by increasing the block size of the input data vector, the 10%-outage-capacities of all the transmit schemes increase.
  - That is due to the increase of the number of the effective transmit eigenmodes.

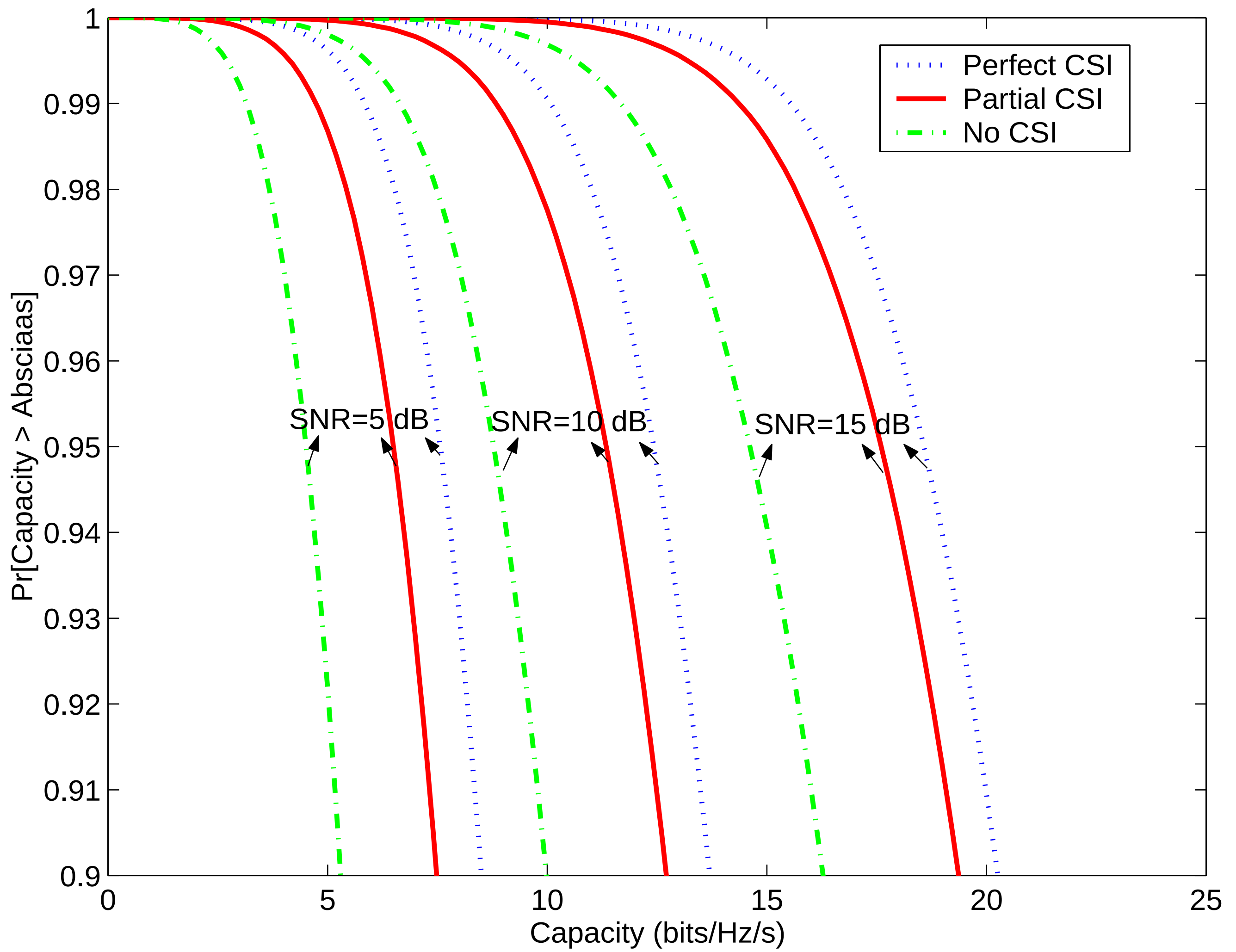



Fig. 2: Capacity complementary cumulative distribution functions of various space-time transmit schemes with the input data temporal block size $N$ being 2 symbols.

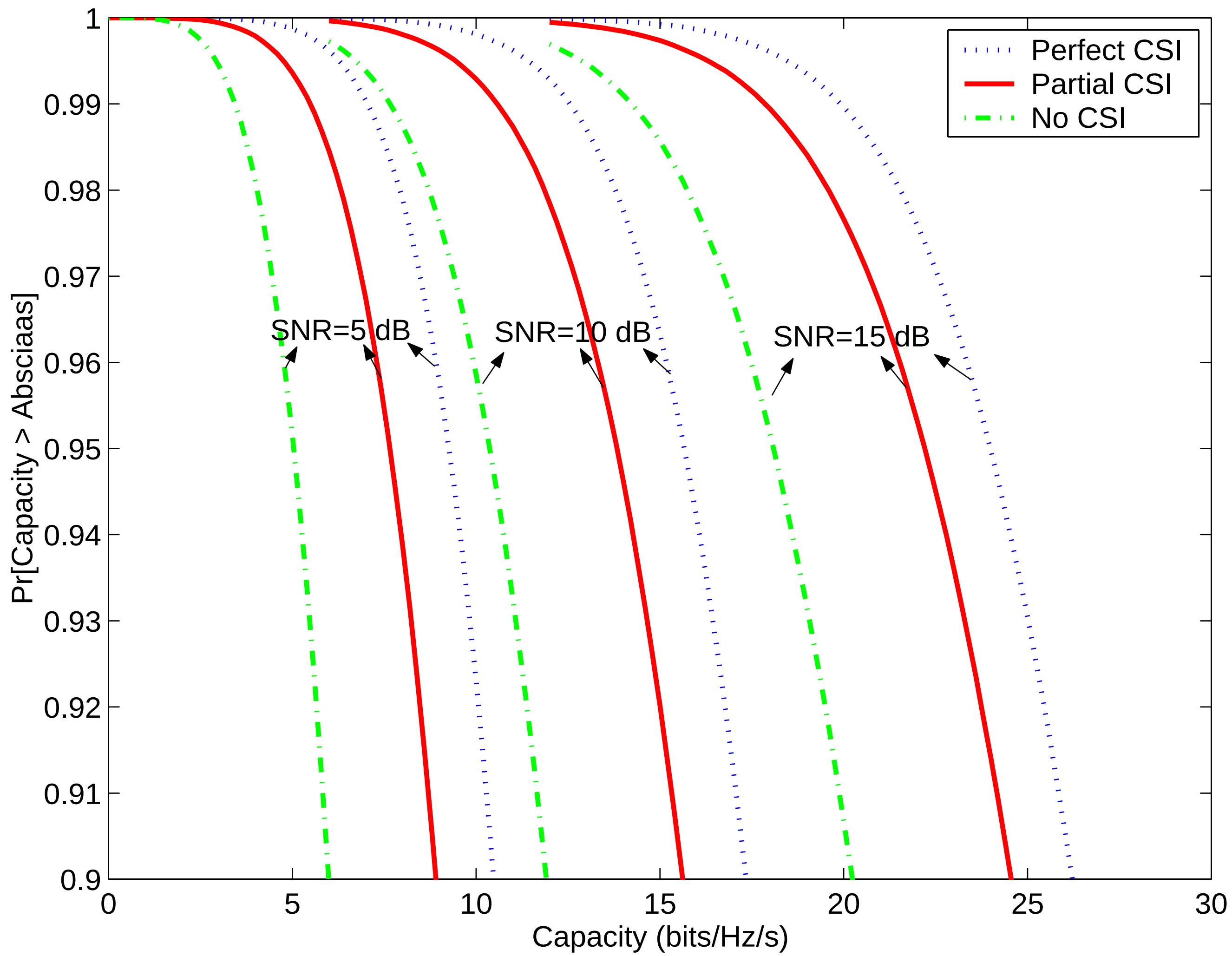



Fig. 3: Capacity complementary cumulative distribution functions of various space-time transmit schemes with the input data temporal block size $N$ being 3 symbols.

## Conclusion

- For the conventional optimum transmit scheme, the algorithm constantly tracks the eigenbeams, which are fast time-varying.
  - High complexity in tracking the fast-varying MIMO CSI and Large amount of CSI feedback.
- By exploiting the wireless multipath channel structure, a new space-time transmit scheme which employs a structure-based water-filling algorithm is proposed.
- Outage capacity as the measure of achievable performance is evaluated to confirm the performance advantage of the proposed transmit scheme.