# Effects of Nonlinear Phase Noise to DPSK Signals

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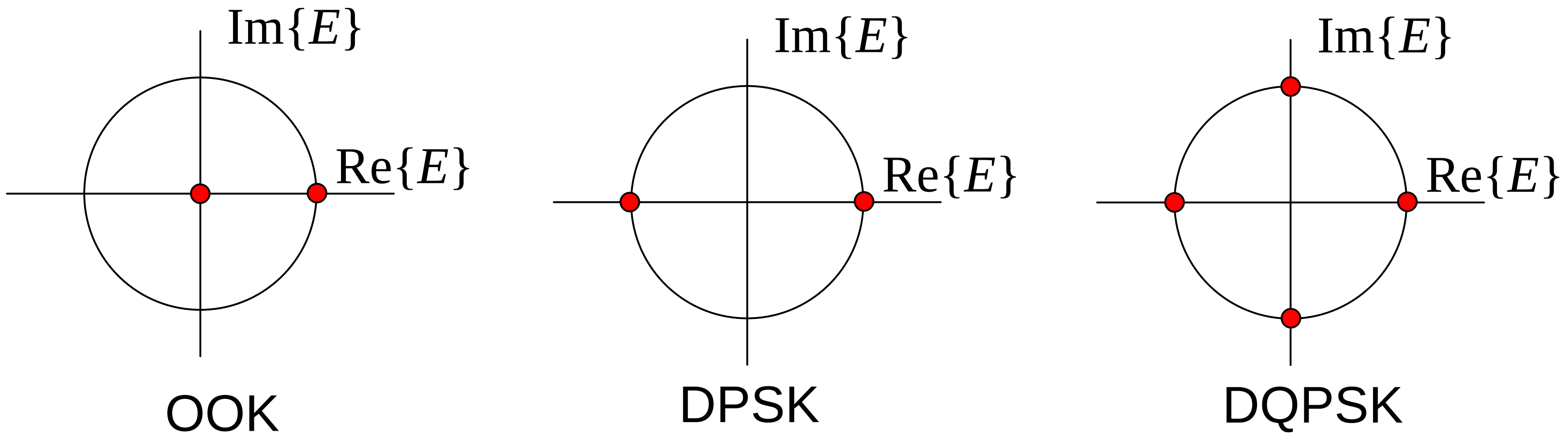
# Outline

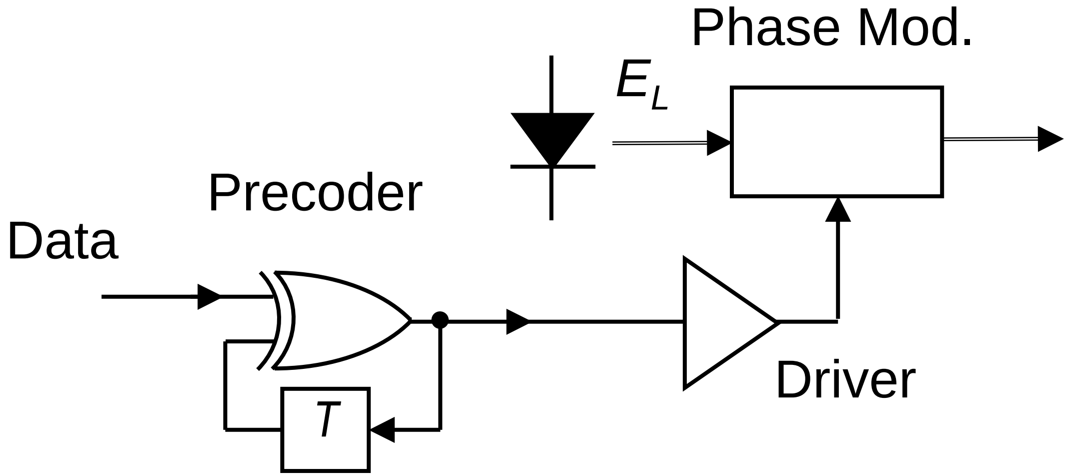
- DPSK Signals
- Nonlinear Phase Noise
- Error Probability
- Linear Compensation
- Summary

# Why DPSK Signals?
## DPSK vs. OOK

- 3-dB improvement on receiver sensitivity
- High tolerance to fiber nonlinearities
  - Low peak power, constant intensity (NRZ), or constant pulse train (RZ)
- Superior spectral efficiency (DQPSK)

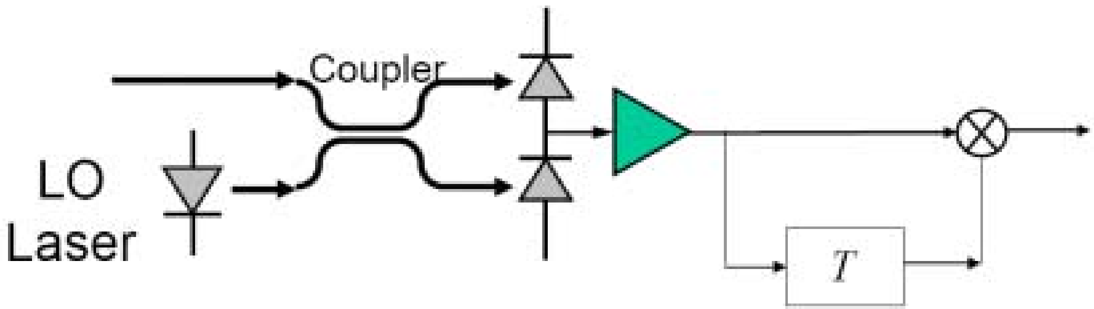


# DPSK Transmitter

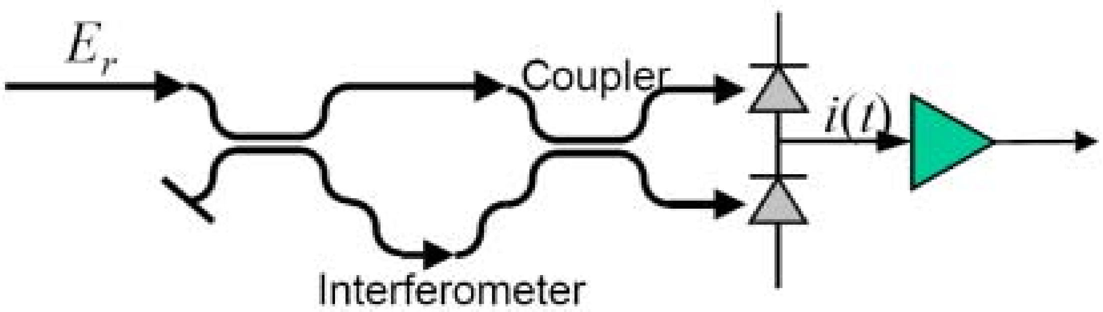



DPSK Transmitter

# DPSK Receiver




Heterodyne Receiver




Direct-Detection Receiver

# Selected Direct-Det. DPSK Demo.

| Rate (Gb/s) | # of channels | Distance (km) | Efficiency (b/s/Hz) | References | Comments |
|---|---|---|---|---|---|
| 42.7 | 64 | 4,000 | 0.4 | A. Gnauck et al. OFC ’02 PD FC2 | First demo |
| 42.7 | 80 | 8,000 | 0.4 | B. Zhu et al. ECOC ’02 PD 4.2 | |
| 42.7 | 40 | 10,000 | 0.4 | C. Rasmussen et al. OFC ’03 PD 18 | |
| 12.3 | 373 | 11,000 | 0.4 | J.-X. Cai et al. OFC ’03 PD22 | |
| 42.7 | 64 | 8,200 | 0.8 | I. Morita & N. Edagawa, ECOC ’03 PD Th.4.3.1 | |
| 10.0 | 96 | 13,000 | 0.3 | J.-X. Cai et al. OFC ’04 PDP 34 | Field trial |
| 42.7 | 6 | 1,600 | 0.4 | A. Gnauck et al. OFC ’04 PDP 35 | 200-km span EDFA only |
| 85.4 | 50 | 4,000 | 1.14 | N. Yoshikane & I. Morita OFC ’04 PDP 38 | DQPSK |

# Nonlinear Phase Noise

- Nonlinear Refractive Index

$$n_r' = n_{r0} + \bar{n}_2(P/A_{\text{eff}})$$

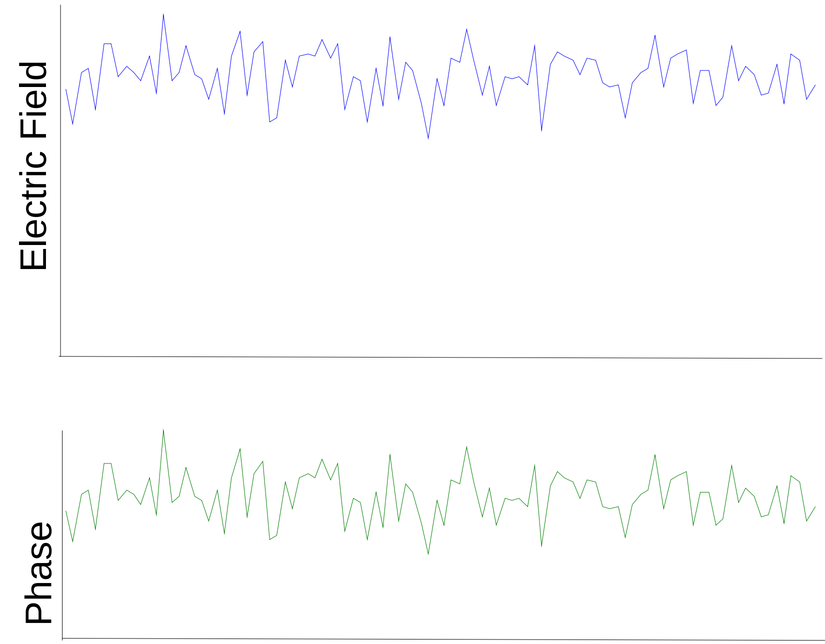
- Nonlinear Phase Shift

$$\phi_{\text{NL}} = \int_0^L \gamma P(z)\mathrm{d}z = \gamma L_{\text{eff}} P$$

$$\gamma = \omega_0 \bar{n}_2/(A_{\text{eff}} c)$$

$$L_{\text{eff}} = (1 - e^{-\alpha L})/\alpha$$

With Amplifier Noise



# Nonlinear Phase Noise

## Math Model

- **Discrete**
  - Linear Electric Field

$$\vec{E}_N = \vec{E}_0 + \vec{n}_1 + \vec{n}_2 + \cdots + \vec{n}_N$$

  - Nonlinear Phase Noise

$$\Phi_{\mathrm{NL}} = \gamma L_{\mathrm{eff}} \Big\{ |\vec{E}_0 + \vec{n}_1|^2 + |\vec{E}_0 + \vec{n}_1 + \vec{n}_2|^2 + \cdots + |\vec{E}_0 + \vec{n}_1 + \cdots + \vec{n}_N|^2 \Big\}$$

Received Electric Field

$$\vec{E}_r = (\vec{E}_0 + \vec{n}_1 + \cdots + \vec{n}_N) \exp(-j\Phi_{\mathrm{NL}})$$

- **Distributed**

$$\vec{E}_N = \vec{\xi}_0 + \vec{b}(1)$$

$$\Phi = \int_0^1 |\vec{\xi}_0 + \vec{b}(t)|^2 \mathrm{d}t$$

$$R_b(t,s) = E\{\vec{b}(s) \cdot \vec{b}(t)\} = \min(t,s)$$

# Nonlinear Phase Noise

## p.d.f. of $\Phi$

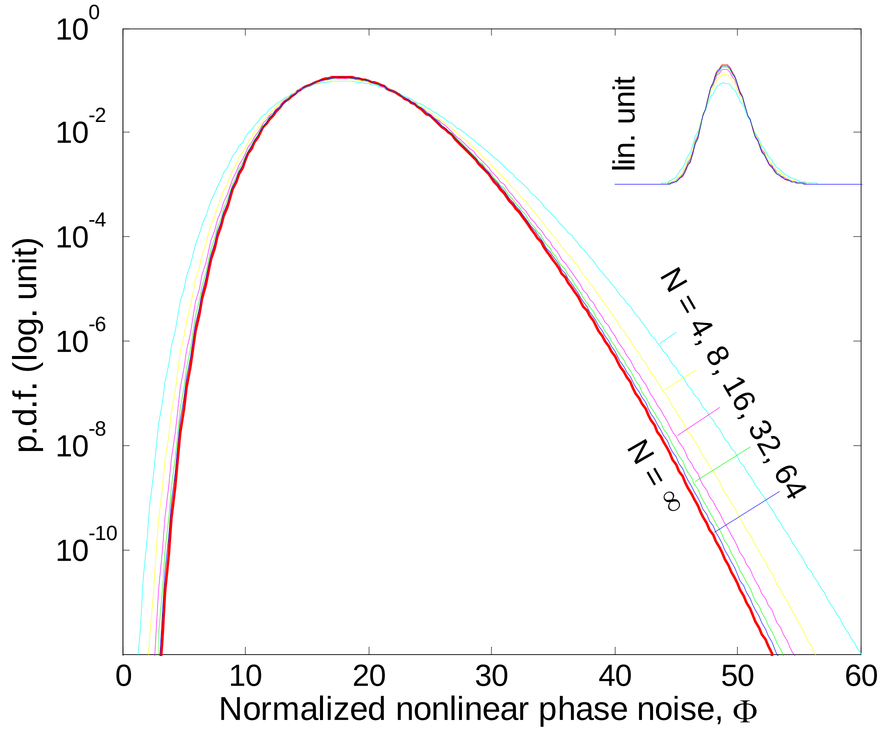
- Characteristic Function

$$\Psi_\Phi(j\nu) = \sec(\sqrt{j\nu}) \exp\left[\rho_s \sqrt{j\nu} \tan(\sqrt{j\nu})\right]$$

- Conversion

$$\Phi_{\mathrm{NL}} = \frac{\langle\Phi_{\mathrm{NL}}\rangle}{\rho_s + \frac{1}{2}} \Phi$$

- More toward positive side

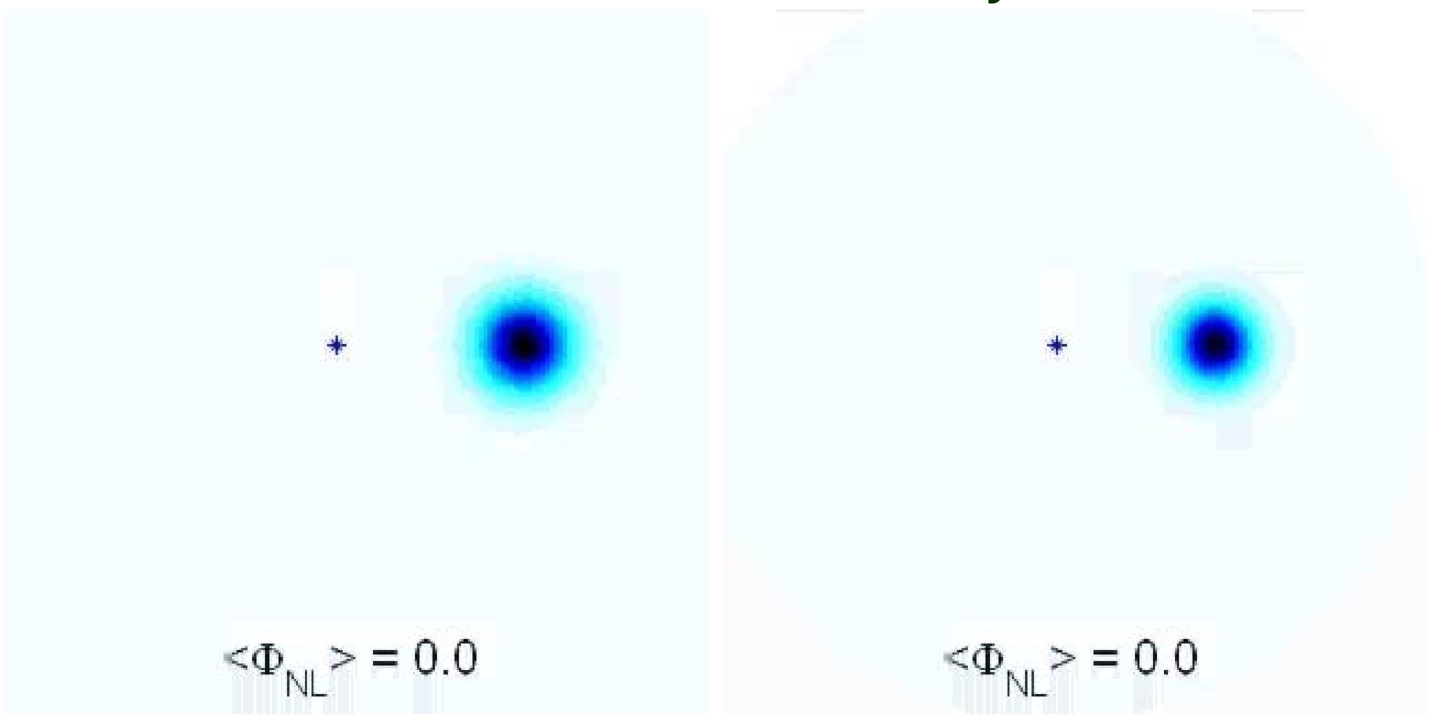



SNR $\rho_s = 18$

K.-P. Ho, Opt. Lett., Aug. 2003 and R. H. Cameron & W. T. Martin, Bull. Am. Math. Soc., 1945.

# Distribution of Signals with Nonlinear Phase Noise

## Simulation vs. Theory




- SNR = 18 (12.6 dB)
- Number of Spans = 32
- Transmitted Signal = (1, 0)

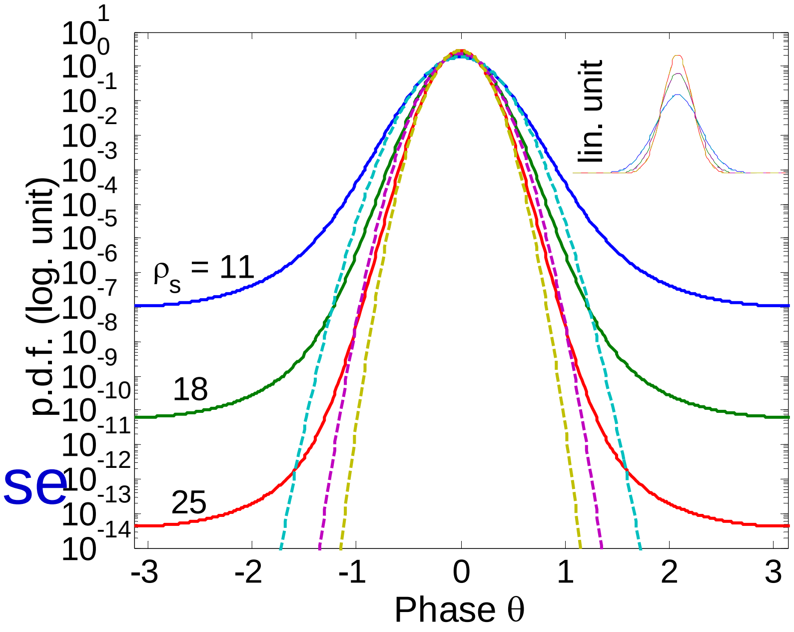
# Distribution of Received Phase

- Received phase

$$\Theta_n - \Phi_{\rm NL}$$

Linear phase noise
(from Gaussian noise)

Nonlinear phase noise

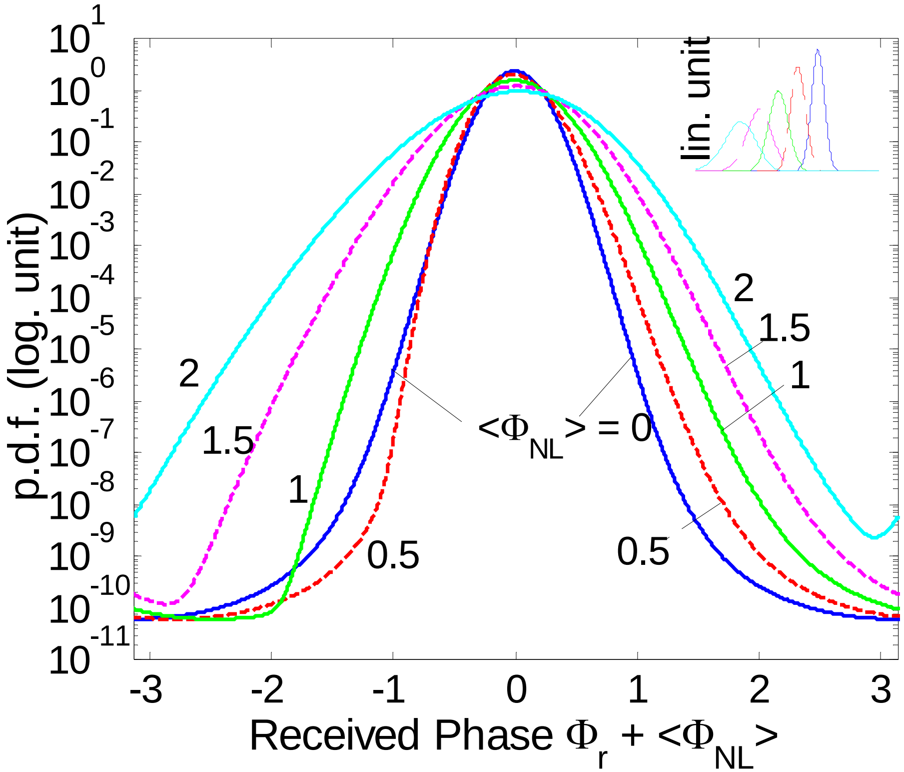



Linear phase noise is
Non-Gaussian!!

# Distribution of the received phase (cont.)

- Small $<\Phi_{NL}>$
  - Positive broadening
- Large $<\Phi_{NL}>$
  - Negative broadening
- Independent
  - Negative broadening

$$<\Phi_{NL}> \approx N\gamma L_{\mathrm{eff}} \, |E_0|^2$$




SNR $\rho_s = 18$

# BER Formulas for DPSK Signal

With Additive Phase Noises, Independent of Gaussian Noise

## With Phase Error

$$p_e = \frac{1}{2} - \frac{\rho_s e^{-\rho_s}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left[ I_k\left(\frac{\rho_s}{2}\right) + I_{k+1}\left(\frac{\rho_s}{2}\right) \right]^2 \cos[(2k+1)\theta_e]$$

## With Laser Phase Noise

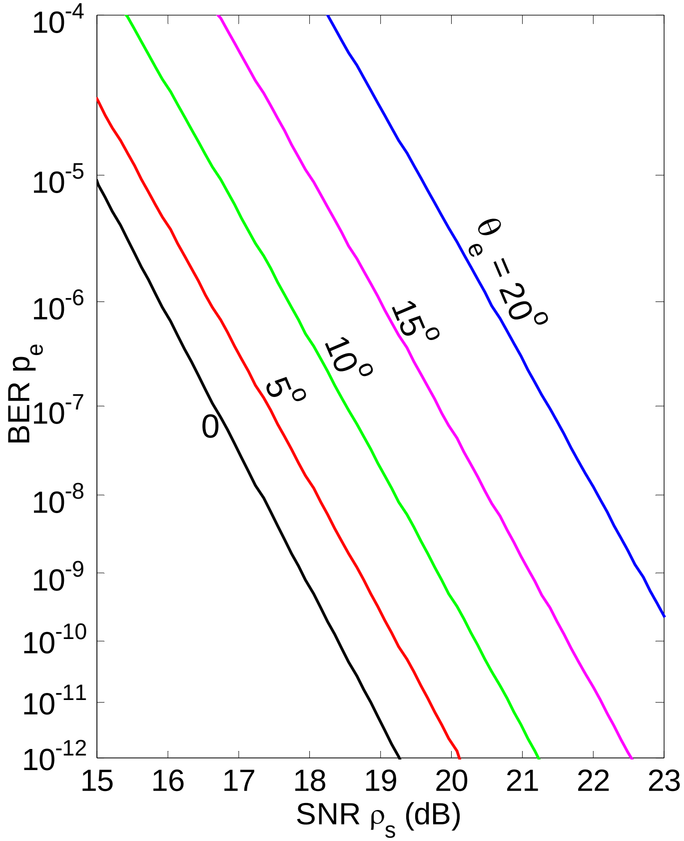
$$p_e = \frac{1}{2} - \frac{\rho_s e^{-\rho_s}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left[ I_k\left(\frac{\rho_s}{2}\right) + I_{k+1}\left(\frac{\rho_s}{2}\right) \right]^2 e^{-\frac{1}{2}(2k+1)^2 \sigma_{\delta\phi}^2}$$

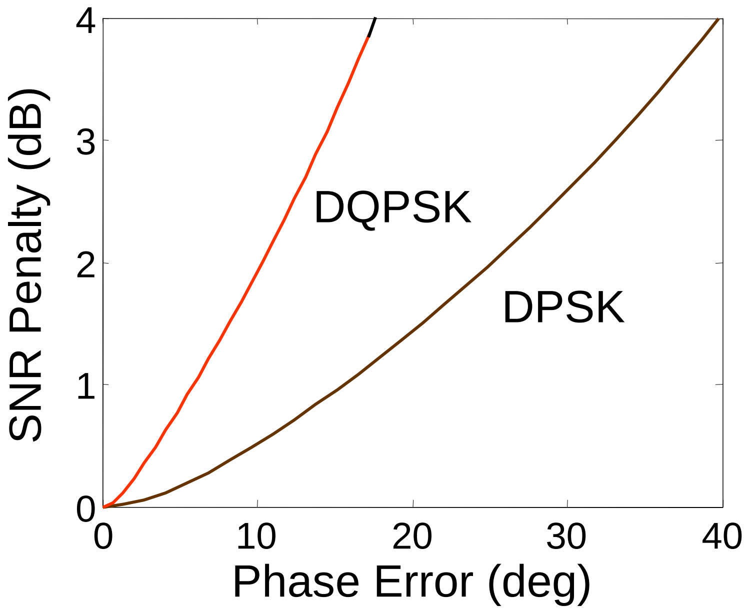
$$\sigma_{\delta\phi}^2 \doteq 2\pi \Delta\nu T$$

## With Nonlinear Phase Noise

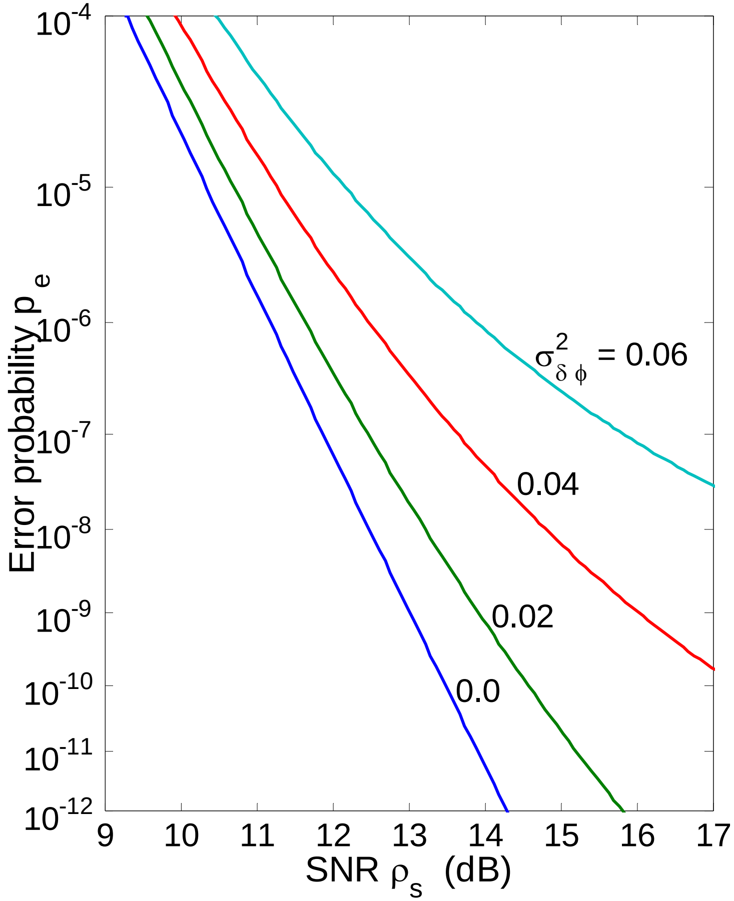
$$p_e \approx \frac{1}{2} - \frac{\rho_s e^{-\rho_s}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left[ I_k\left(\frac{\rho_s}{2}\right) + I_{k+1}\left(\frac{\rho_s}{2}\right) \right]^2 \left| \Psi_\Phi \left[ \frac{(2k+1) <\Phi_{\mathrm{NL}}>}{\rho_s + 1/2} \right] \right|^2$$

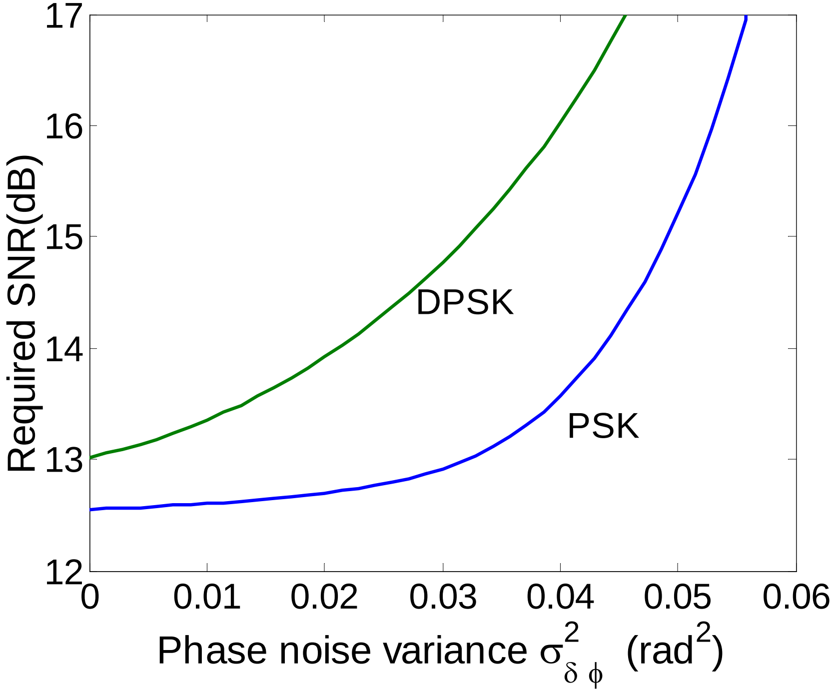
# DPSK Signal with Phase Error



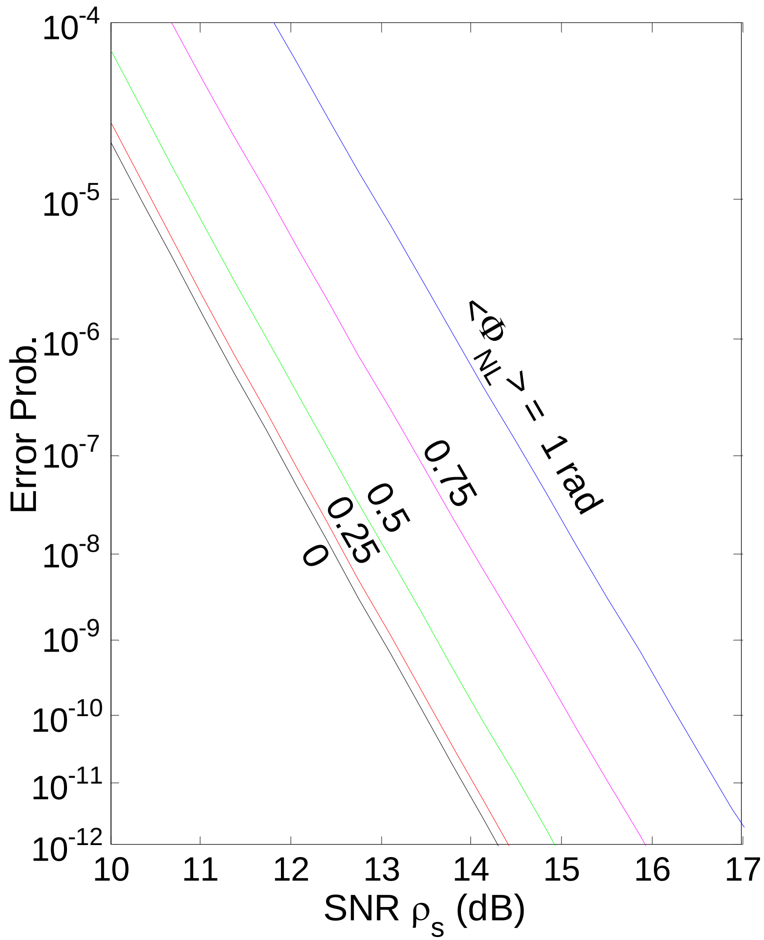



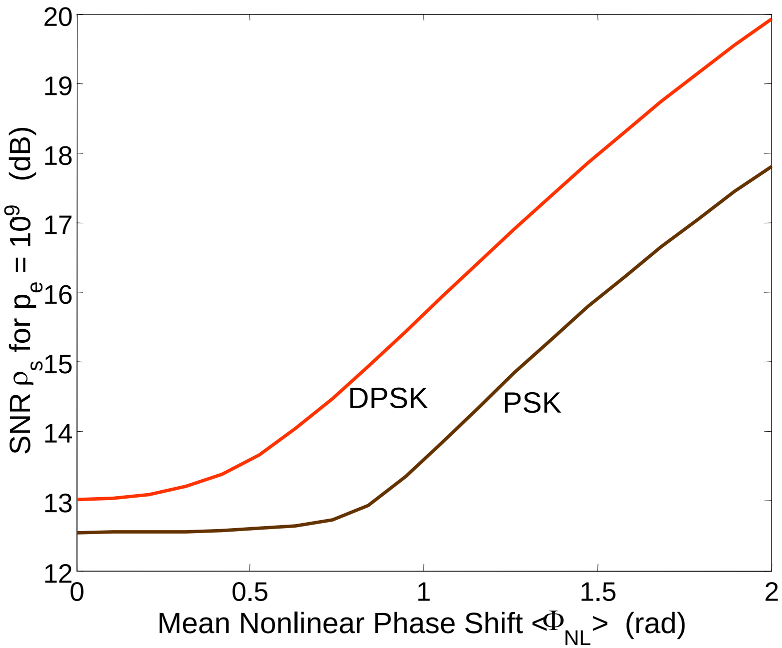
# DPSK Signal with Laser Phase Noise



# DPSK Signal with Nonlinear Phase Noise (Approx.)



# Exact Error Probability

- Independent

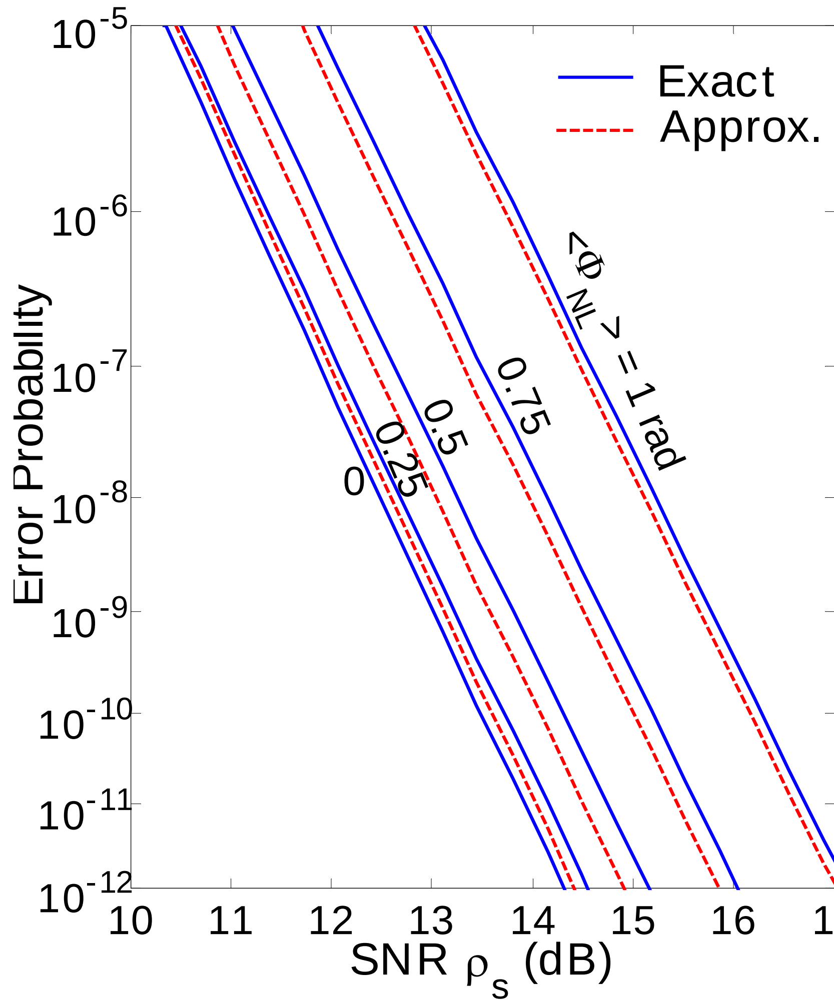
$$p_e \approx \frac{1}{2} - \frac{\rho_s e^{-\rho_s}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \left[ I_k\left(\frac{\rho_s}{2}\right) + I_{k+1}\left(\frac{\rho_s}{2}\right) \right]^2 \left| \Psi_\Phi \left[ \frac{(2k+1) <\Phi_{\mathrm{NL}}>}{\rho_s + 1/2} \right] \right|^2$$

- Dependent

$$p_e = \frac{1}{2} - \frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^k \left| \lambda_k e^{-\lambda_k} \right|}{2k+1} \left| I_k\left(\frac{\lambda_k}{2}\right) + I_{k+1}\left(\frac{\lambda_k}{2}\right) \right|^2 \left| \Psi_\Phi \left[ \frac{(2k+1) <\Phi_{\mathrm{NL}}>}{\rho_s + \frac{1}{2}} \right] \right|^2$$

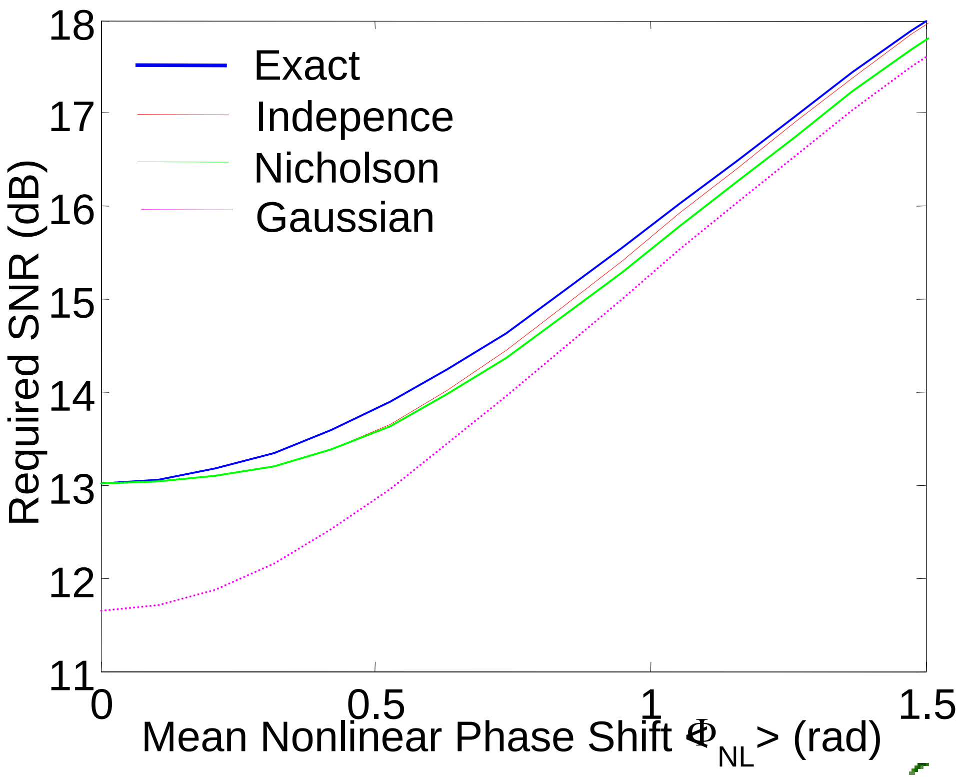
$$\lambda_k = \frac{2 \left[ \frac{j(2k+1) <\Phi_{\mathrm{NL}}>}{\rho_s + 1/2} \right]^{1/2}}{\sin \left\{ 2 \left[ \frac{j(2k+1) <\Phi_{\mathrm{NL}}>}{\rho_s + 1/2} \right]^{1/2} \right\}} \rho_s.$$

# Exact Error Probability (DPSK Signals)



# SNR Penalty
## A Comparison

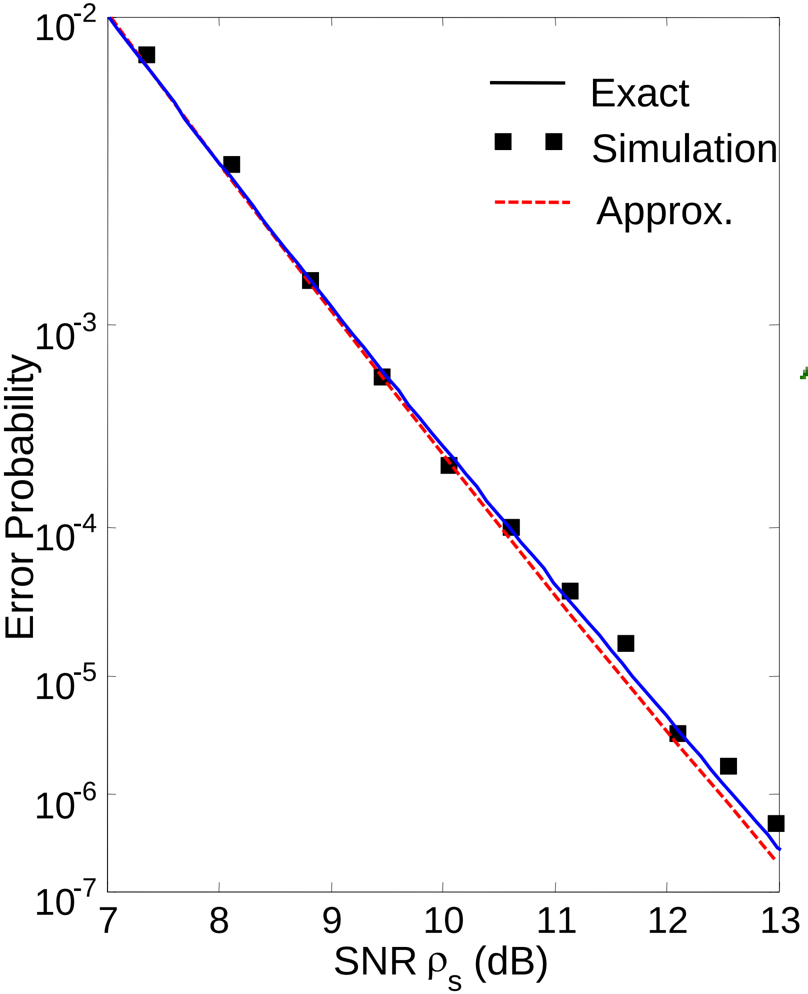
- All approximated models underestimate the BER and SNR penalty.
- The independence model underestimates up to 0.23 dB.
- The Nicholson model underestimates up to 0.27 dB.
- Q-factor model is complete failure except at high nonlinear phase noise



# Error Probability
## Simulation

$\langle \Phi_{NL} \rangle$ = 0.71 rad
32 spans

DPSK Signals



# Nonlinear Phase Noise
## Yin-Yang Detector

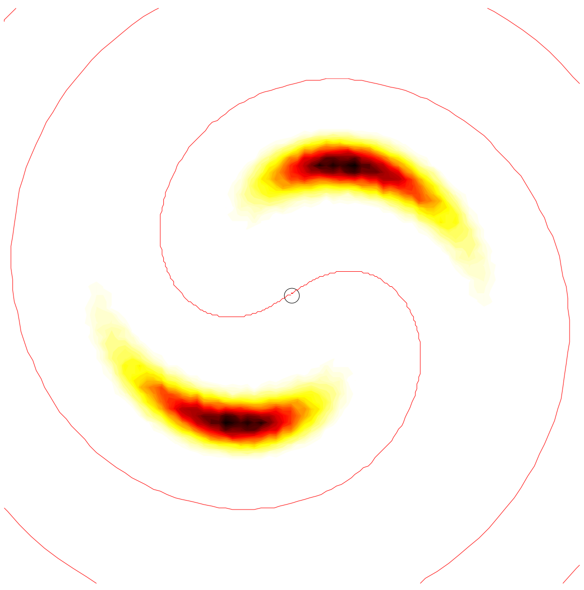
$<\Phi_{NL}>$ = 2 rad
32 spans

The boundary is

$$\theta + \alpha r^2 = \theta_0$$

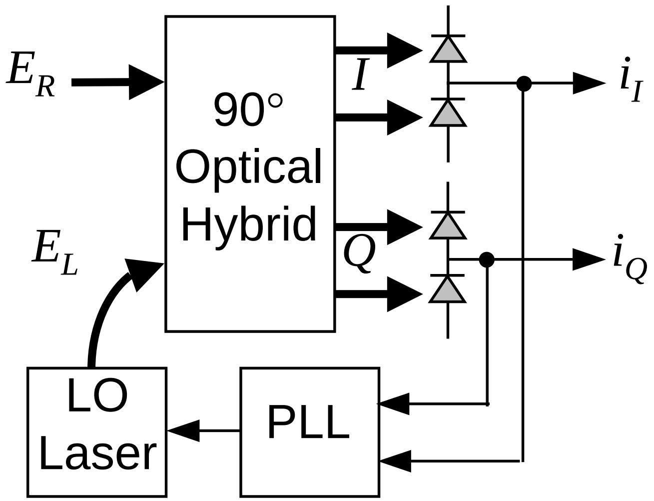
A compensator of

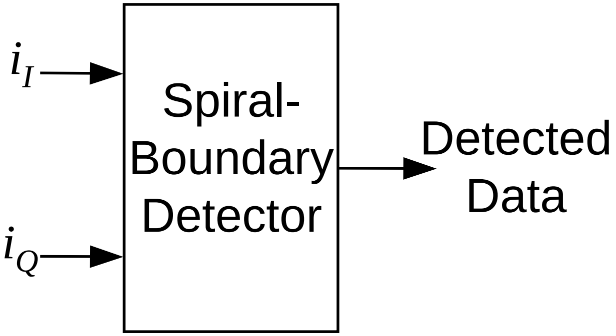
$$\phi_r + \alpha r^2$$

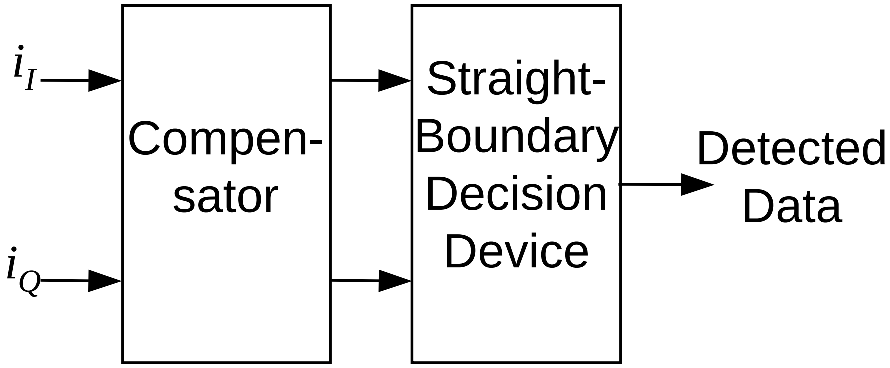


$<\phi_{NL}> = 2.0$

# Curved Detector or Compensator

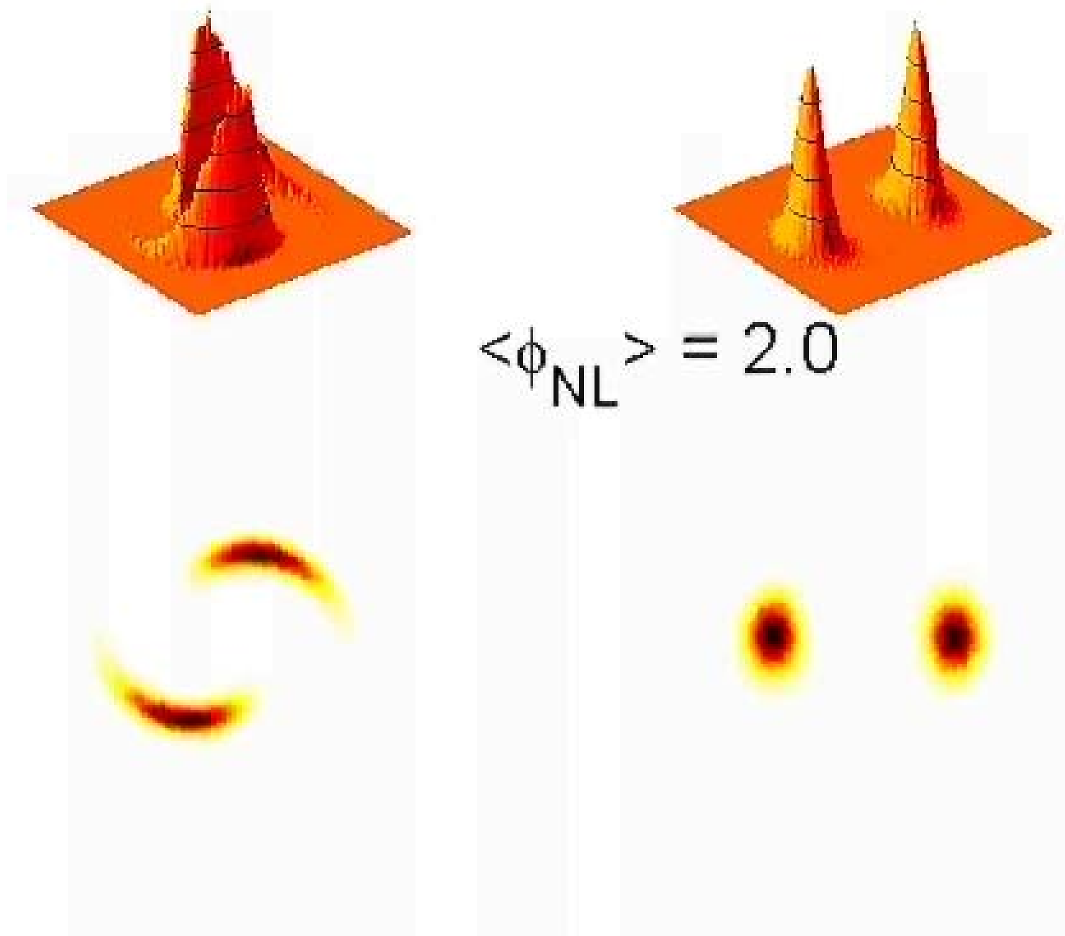


# Nonlinear Phase Noise

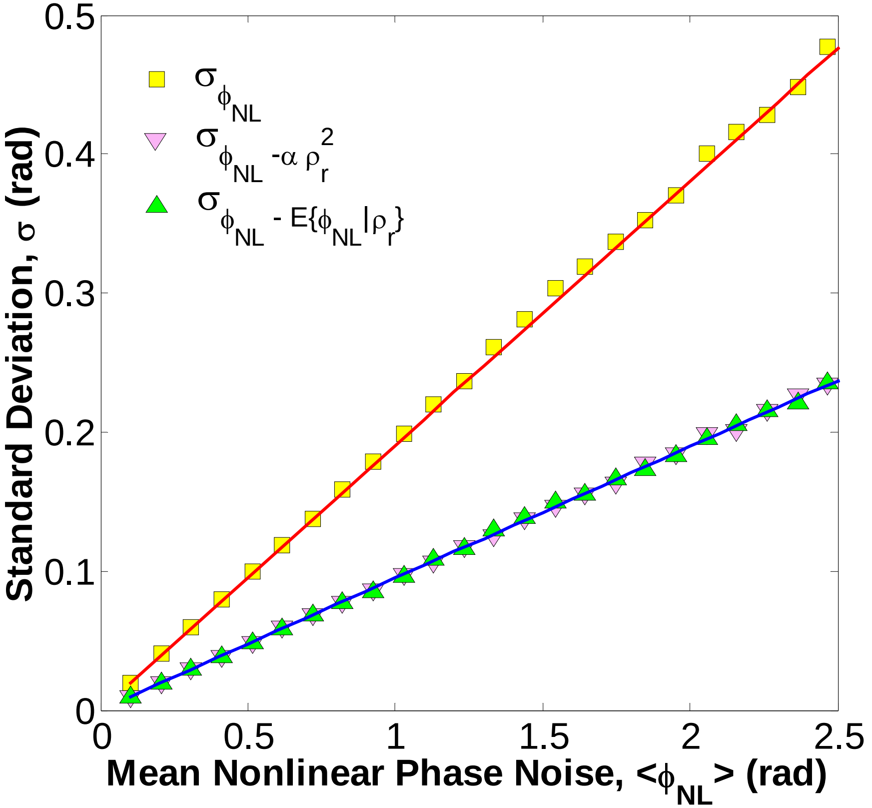
## Linear Compensator




$\phi_r - \alpha r^2$

# Linear Compensator
## Variance of Nonlinear Phase Noise

- Linear compensator
  - $\phi_r + \alpha r^2$
- Nonlinear compensator
  - $\phi_r + E\{\phi_{NL}|r\}$
- STD is halved
- Linear and nonlinear compensator perform the same
- Double transmission distance?

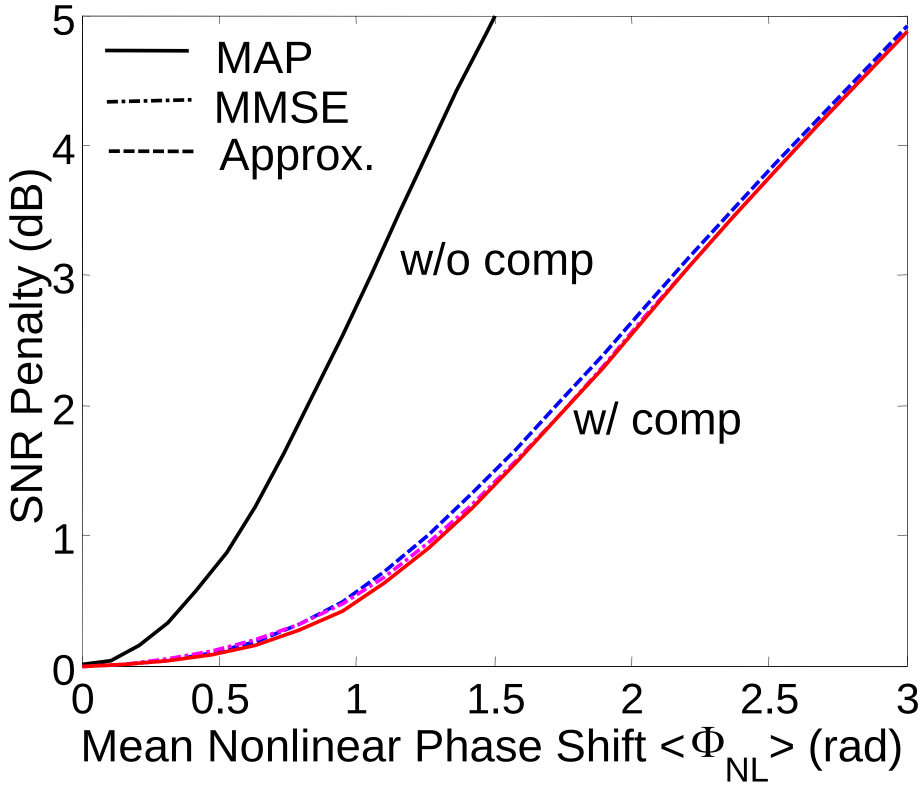


# Linear Compensator

## For DPSK signals

- Exact BER is derived
- MMSE compensator is derived
- MAP compensator is found numerically

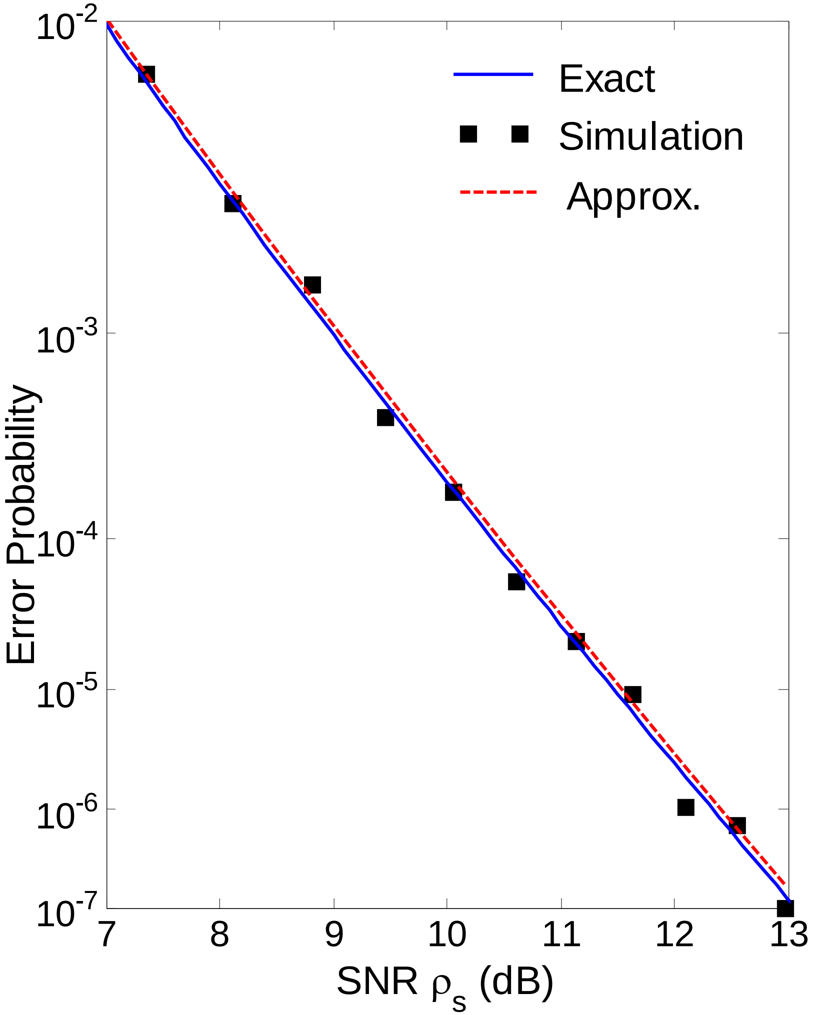
$$<\Phi_{\mathrm{NL}}> \approx N\gamma L_{\mathrm{eff}} \mid E_0 \mid^2$$



# Error Probability with Linear Compensation

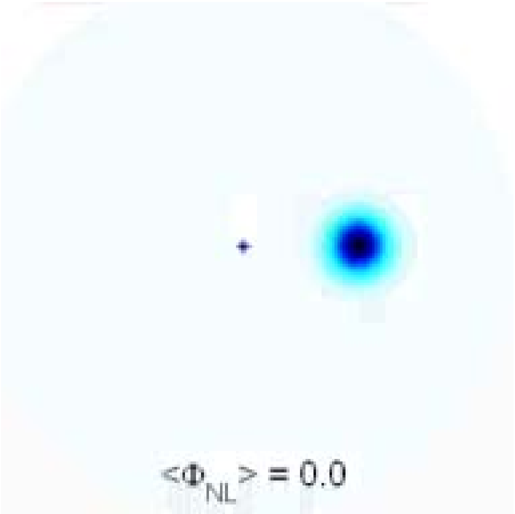
## Simulation

$\langle\Phi_{NL}\rangle = 1.41$ rad
32 spans



# Summary

- Model of nonlinear phase noise
  - Finite number of spans
  - Asymptotic model for many spans
  - Joint distribution of amplitude and phase
- Error probability for DPSK signal
  - Approximation and exact formulas
- Linear Compensation
  - MMSE & MAP compensation



# References

Most of the materials are already in or will be in
http://arxiv.org/physics/0303090
and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