Modelocked Fiber Lasers and Quantum Solitons

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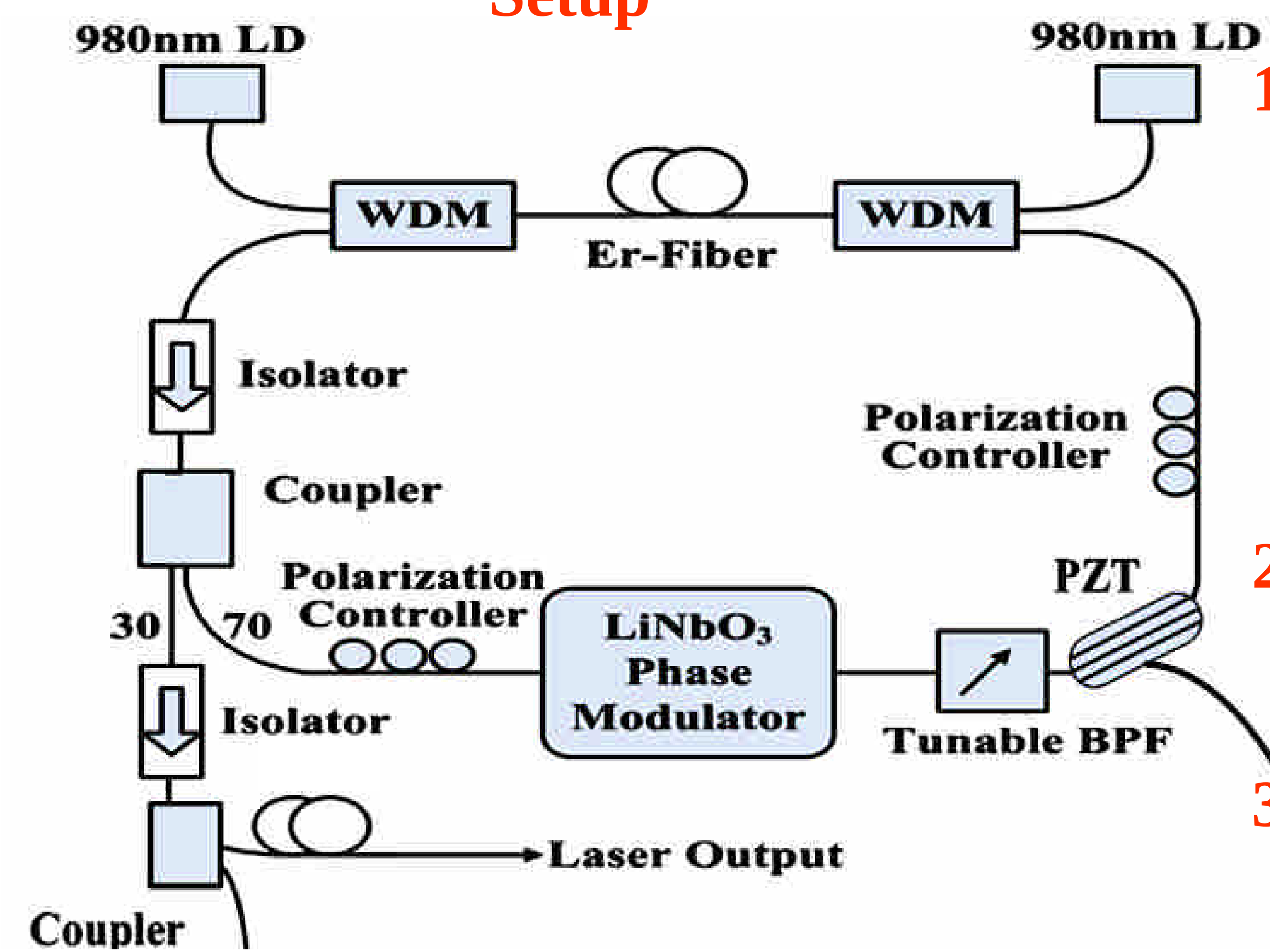
[Outline]

1. Asynchronous modelocked Er-fiber soliton laser at 10GHz
2. Bound soliton fiber laser at 10GHz
3. Laser dynamics and noise properties
4. Possible applications

Fiber Soliton Laser Configuration

Setup



Features:

1. 10-40GHz, 500-800fs Pulse train directly from the laser.
   (Tens mW at 1550nm)
2. Active harmonic modelocking +Passive P-APM.
3. Asynchronous and Bound soliton

W.-W. Hsiang, C.-Y. Lin, M.-F. Tien, and Y. Lai, Opt. Lett. 30, 2493-2495 (2005) .

Solitons in optical fibers

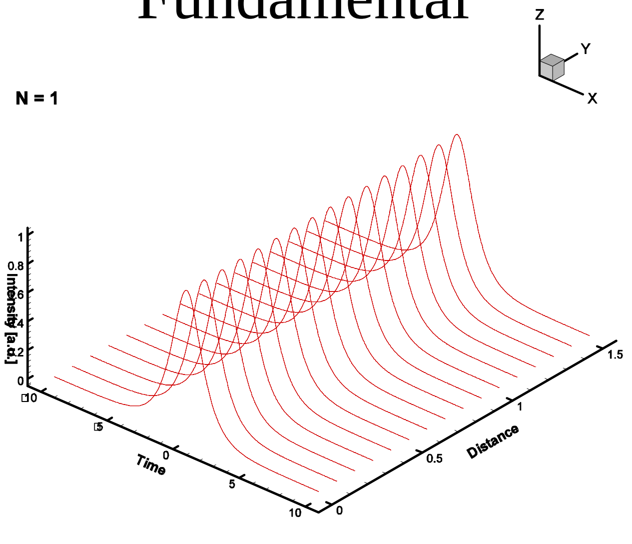
Nonlinear Schroedinger equation (NLSE):

$$i\frac{\partial}{\partial z}U(z,t) = -\frac{1}{2}\frac{\partial^2}{\partial t^2}U(z,t) - |U(z,t)|^2\, U(z,t)$$

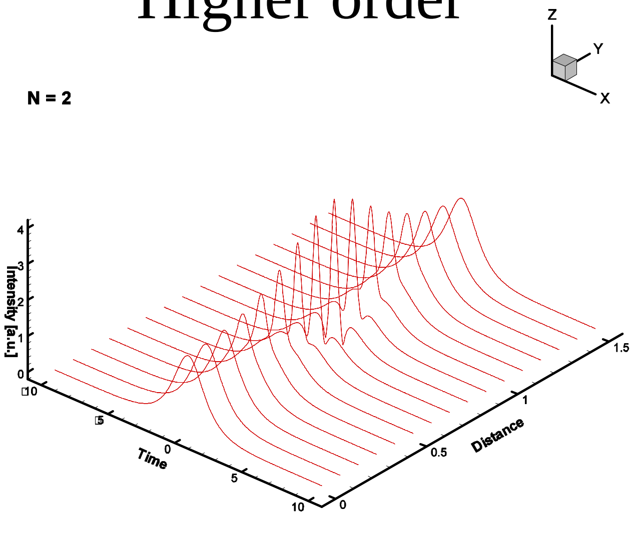
Fundamental solitons:

$$U(z,t) = \frac{n_0}{2}\exp[i\frac{n_0^2}{8}z + i\theta_0]\,\mathrm{sec}h[\frac{n_0}{2}t]$$

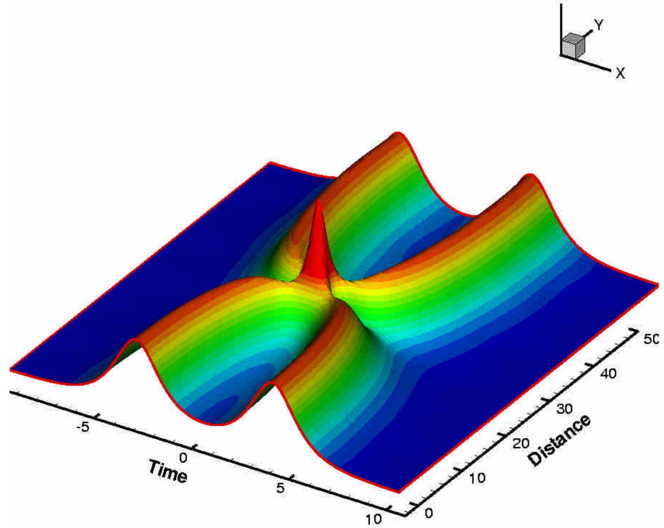
Fundamental



Higher order



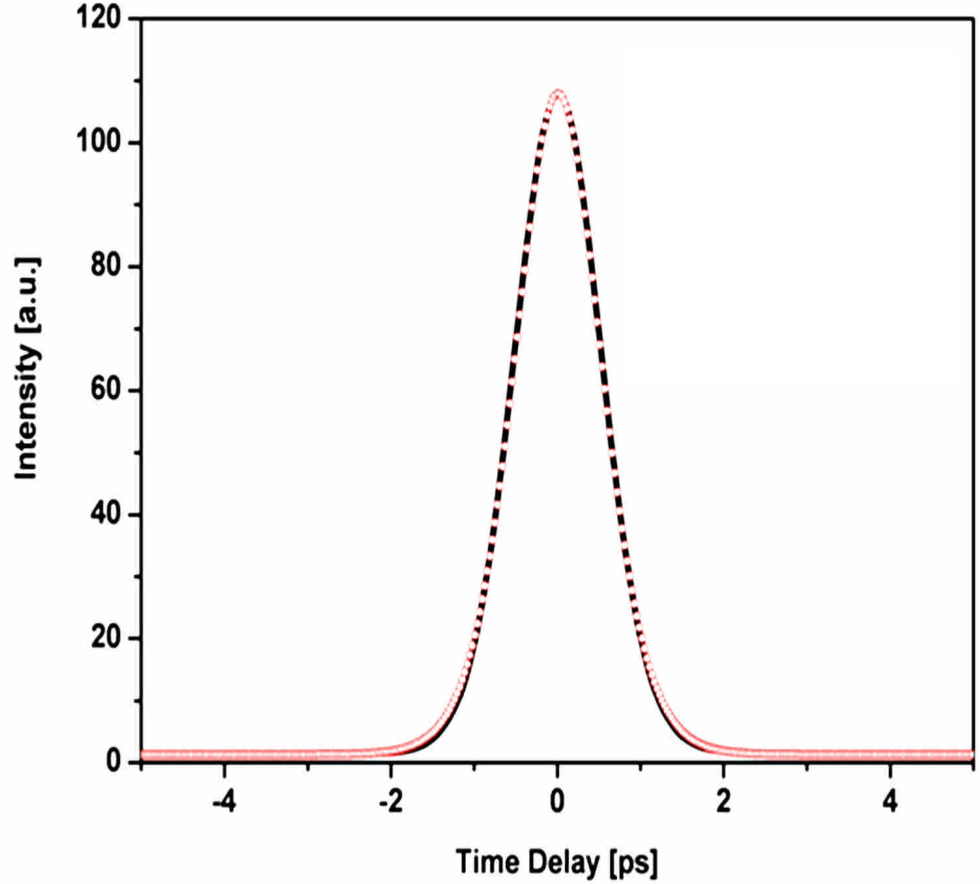
Soliton interaction



Output Laser pulses

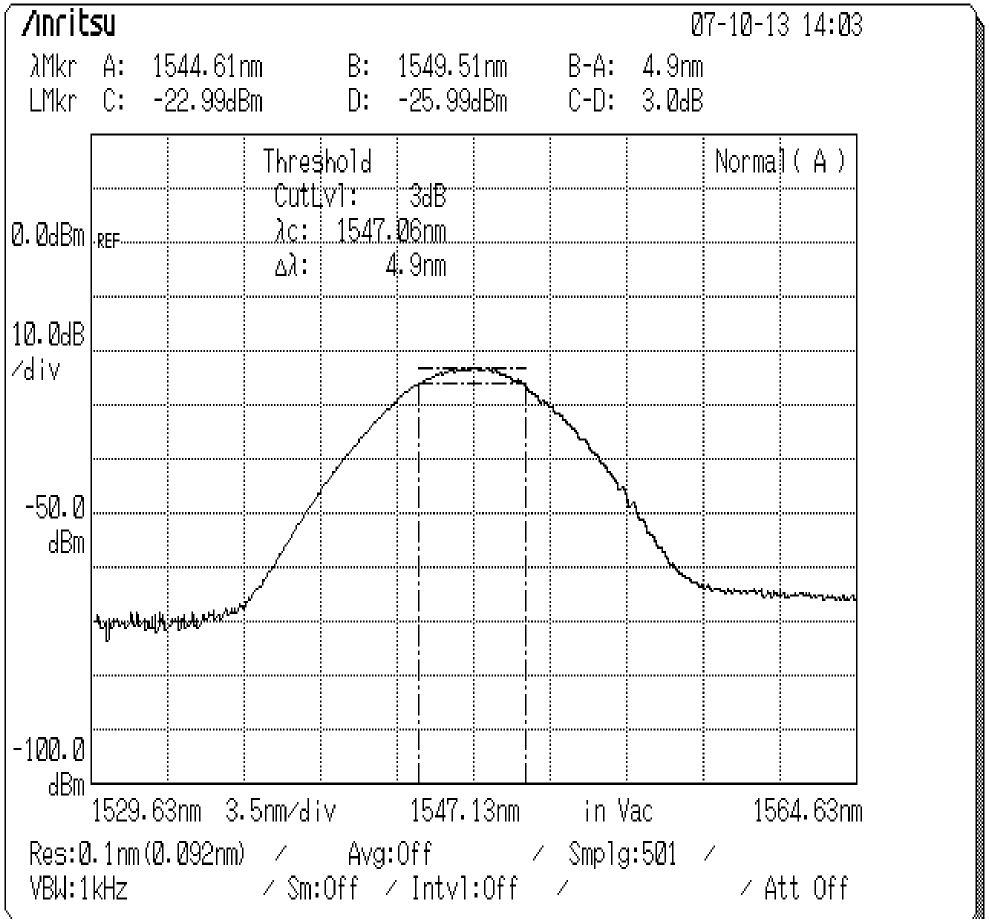
Case 1: Autocorrelation

10GHz 800fs



Case 2 : Optical Spectrum

10GHz, 4.9nm bandwidth



10-40GHz repetition rate, 500-800fs transform-limited pulsewidth

Different Modes of Operation

Four operation modes on the same laser configuration

1. Synchronous active harmonic modelocked
2. Synchronous soliton harmonic modelocked
3. Asynchronous soliton harmonic modelocked
4. Synchronous bound soliton harmonic modelocked

Harmonic modelocked:

Multiple (over 1000) pulses in the cavity

Synchronous:

Modulation frequency = cavity harmonic frequency

Asynchronous:

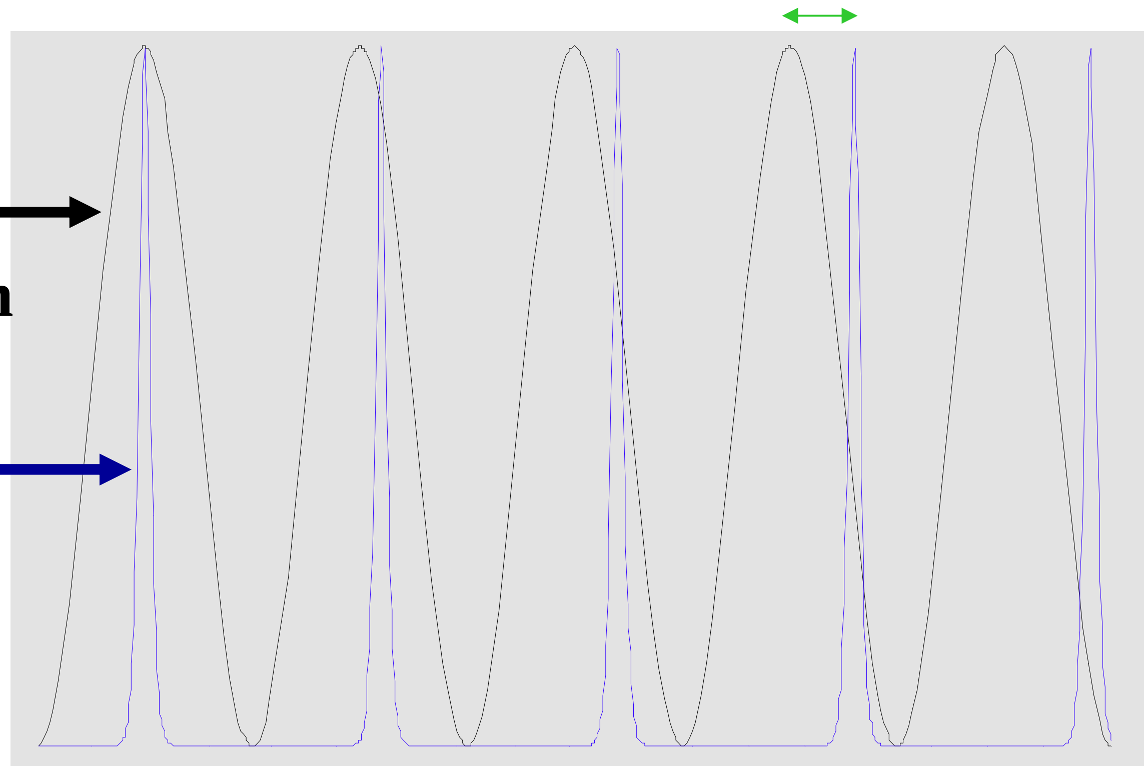
Modulation frequency $\neq$ cavity harmonic frequency (with few tens kHz frequency deviation)

Asynchronous Soliton Modelocking (I)

Timing walk-off

EO Phase Modulation

Soliton Pulse train



time

Slow Periodic Frequency Shift of the Solitons!!

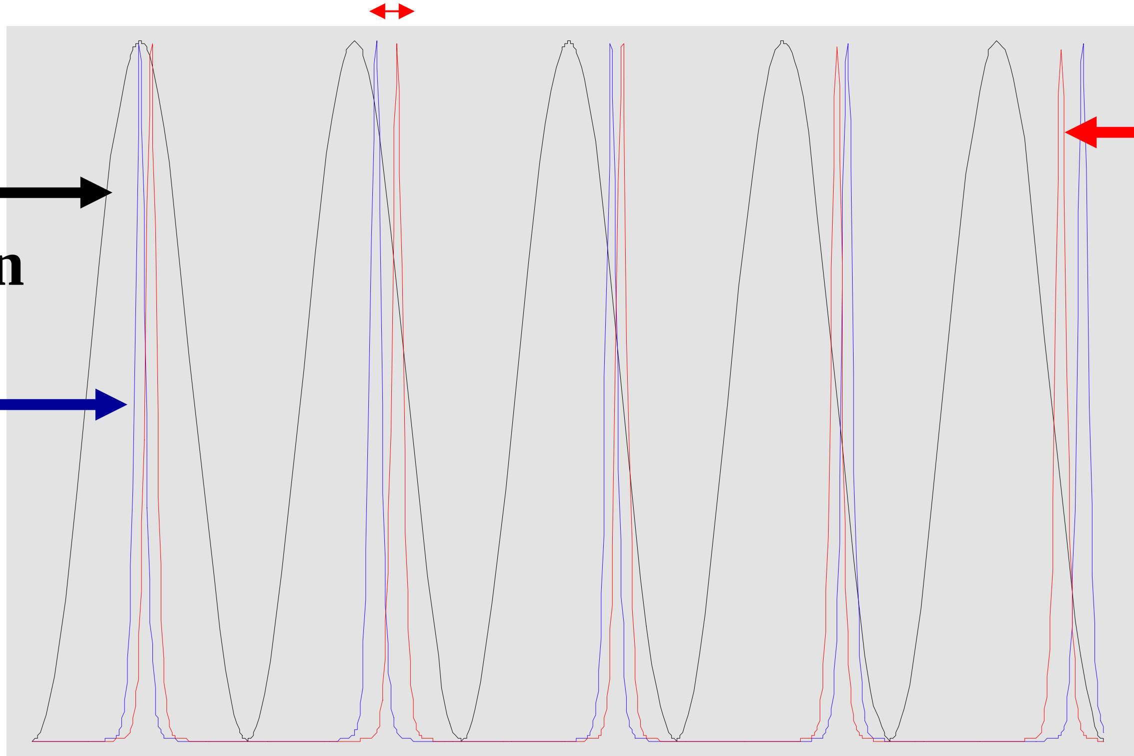
Modulation frequency ≠ cavity harmonic frequency (with few tens kHz frequency deviation)

Asynchronous Soliton Modelocking (II)

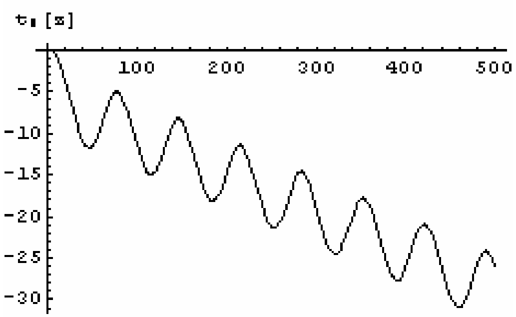
Slow Periodic Pulse Timing oscillation

Phase Modulation

Linear Timing Walk-off



Actual Soliton Pulse train



time

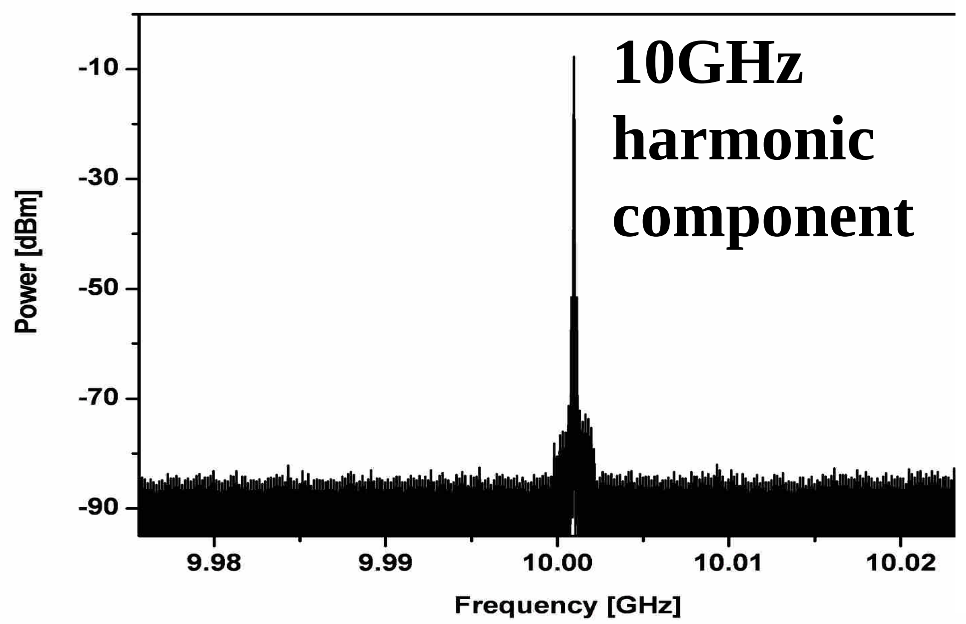
Periodic frequency shift ➔ Periodic Timing oscillation

dispersion

Asynchronous Soliton Modelocking (III)

Laser output measured by an electronic spectrum analyzer.

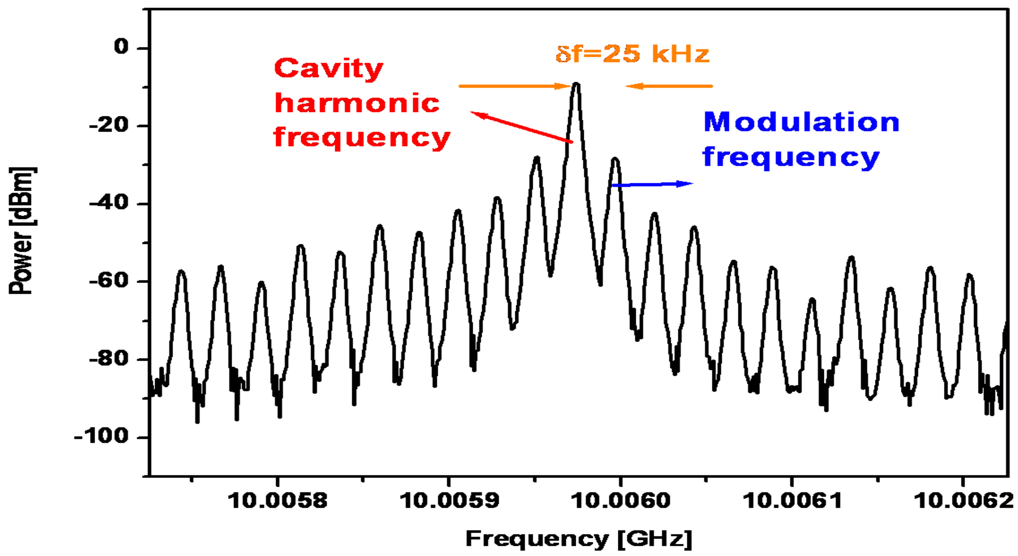
RF spectrum (I)



SMSR > 70dB

RF spectrum (II)

25kHz sidebands



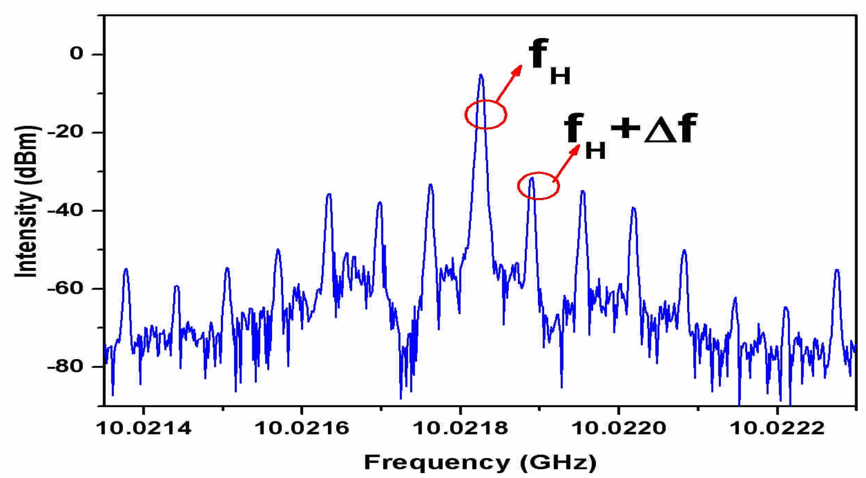
Experimental Determination of Timing Oscillation

$$I(t) \propto |u(t)|^2 \otimes \sum_{m=-\infty}^{m=\infty} \delta[t - mT_H + s_0 \sin(\Delta\Omega m T_H)]$$

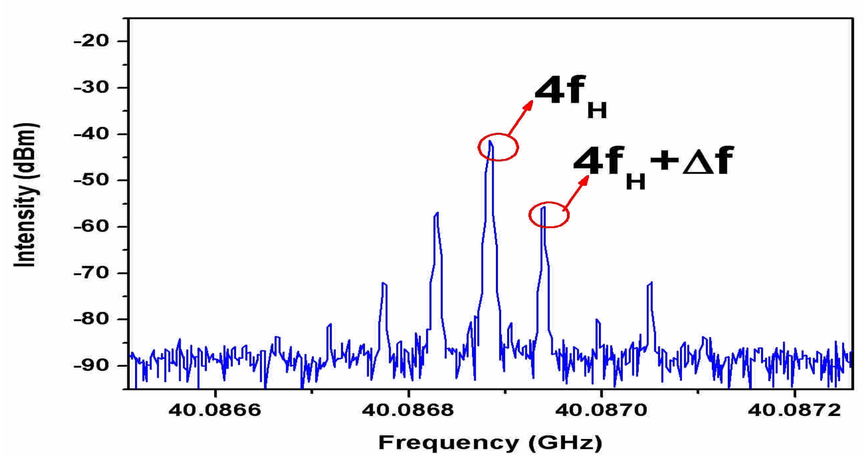
$$F\{I(t)\} = F\{|u(t)|^2\} F\left\{\sum_{m=-\infty}^{m=\infty} \delta[t - mT_H + s_0 \sin(\Delta\Omega m T_H)]\right\}$$

$$\propto F\{|u(t)|^2\}\left[\sum_{n=-\infty}^{n=\infty} J_n(\omega s_0) \sum_{m=-\infty}^{m=\infty} \delta(\omega - m\Omega_H + n\Delta\Omega)\right]$$

Ratio of 0 and 1st order sub-components= $\dfrac{J_0(\omega_H s_0)}{J_1[(\omega_H + \Delta\omega)s_0)]} \cong \dfrac{J_0(\omega_H s_0)}{J_1(\omega_H s_0)}$



ω



Magnitude of Timing Oscillation

Table 1: Comparison of the ratios of the RF intensity at mf_H to that at $mf_H + \Delta f$ from $m = 1$ to $m = 4$.

	RF intensity I_A at mf_H (dBm)	RF intensity I_B at $mf_H + \Delta f$ (dBm)	$I_A - I_B$ (dBm)	$20Log\left[\frac{J_0(mx)}{J_1(mx)}\right]$ (dB) $x = 0.09474$
m=1	-5.25	-31.72	26.48	26.48
m=2	-36.21	-55.23	19.02	20.43
m=3	-37.63	-53.4	15.77	16.86
m=4	-41.46	-55.75	14.29	14.29

$\mathbf{x=2\pi}\ s_0 f_H$ **=0.09474** → s_0 = **1.5ps**

f_H**=10GHz**

To be presented at CLEO2008

Laser Dynamics of Asynchronous Modelocking (I)

Maser equation model:

$$\frac{\partial U}{\partial T} = \left(\frac{g_0}{1 + \frac{\int_{-\infty}^{\infty} |u(T,t)|^2 dt}{E_S}} - l_0 \right) U + i\, s_m \cos(\omega_m t)\, U + (d_r + i\, d_i) \frac{\partial^2 U}{\partial t^2} + (k_r + i\, k_i)\, U^* U^2$$

Gain **Loss** **Phase modulation** **Filtering** **Dispersion** **APM** **SPM**

T: number of roundtrips (large time scale)
t: time (small time scale)

Laser Dynamics of Asynchronous Modelocking (II)

Maser equation:

$$\frac{\partial U}{\partial T} = \left(\frac{g_0}{1 + \frac{\int_{-\infty}^{\infty} |u(T,t)|^2 dt}{E_S}} - l_0 \right) U + i\, s_m \cos(\omega_m t)\, U \; + (d_r + i\, d_i)\, \frac{\partial^2 U}{\partial t^2} + (k_r + i\, k_i)\, U^* \, U^2$$

Solution ansatz:

$$U(T,t) = a(T) \operatorname{sech}\left[\frac{t - t_0(T)}{\tau(T)}\right] \exp\left[ib(T) \ln\left[\operatorname{sech}\left[\frac{t - t_0(T)}{\tau(T)}\right]\right] + i\omega(T)(t - t_0(T)) + i\theta(z)\right]$$

Frequency

$$\frac{d\,\omega}{\mathrm{dT}} = -s_m\, \omega_m \sin[\omega_m\, t_0(T)] - \frac{4\, d_r \left(1 + b(T)^2\right) \omega(T)}{3\, \tau(T)^2}$$

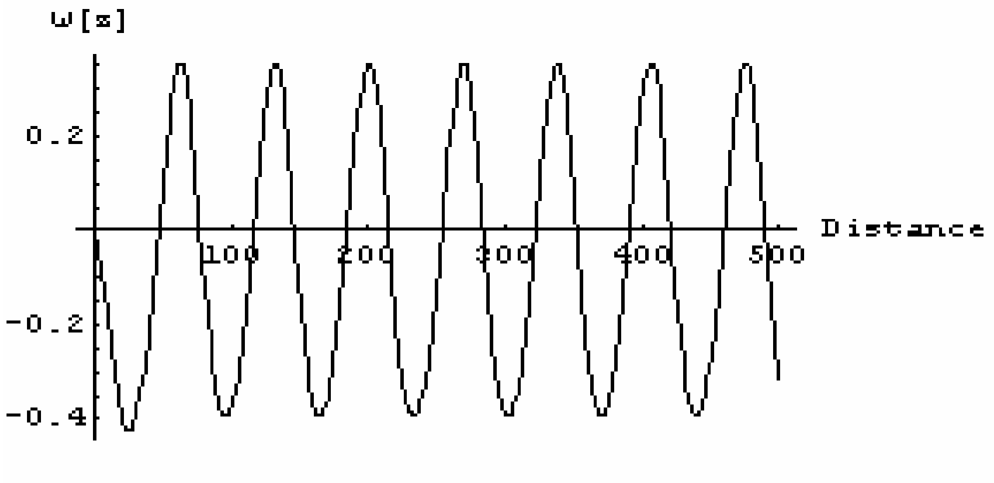
Position

$$\frac{d\,t_0}{\mathrm{dT}} = 2\, d_i\, \omega(T) + 2\, d_r\, b(T)\, \omega(T) + R_{\mathrm{delay}}$$

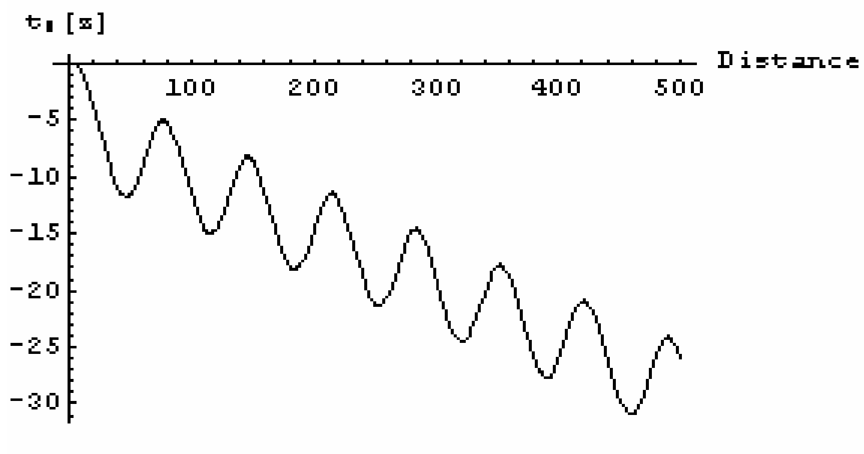
Fixed delay

Laser Dynamics of Asynchronous Modelocking (III)

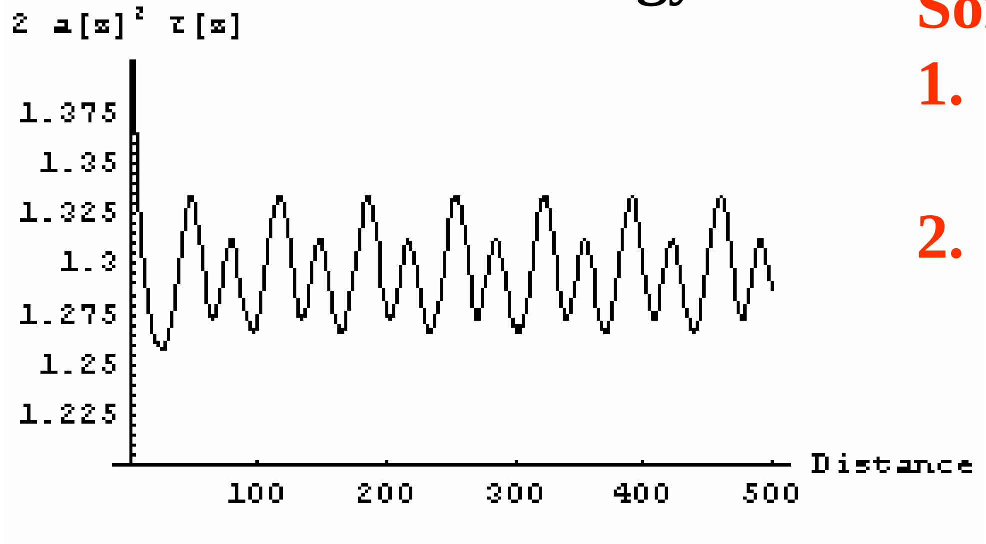
Frequency



Timing position



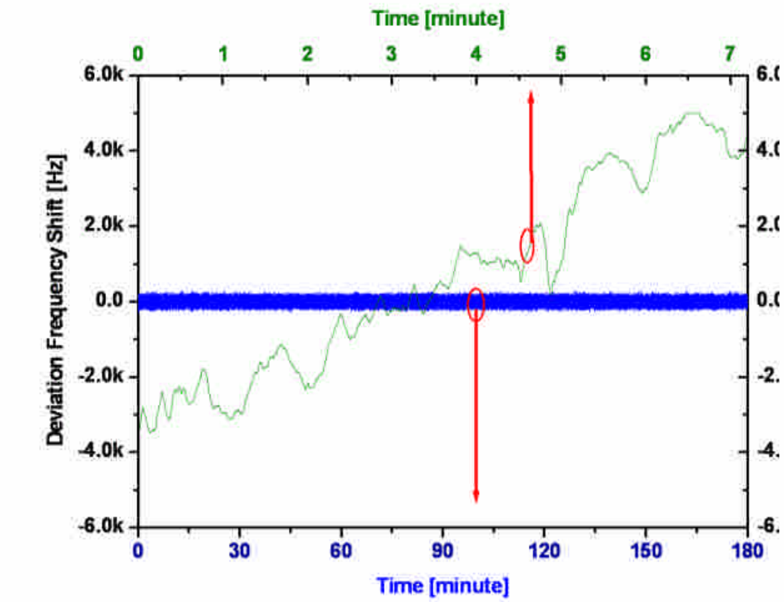
Pulse Energy

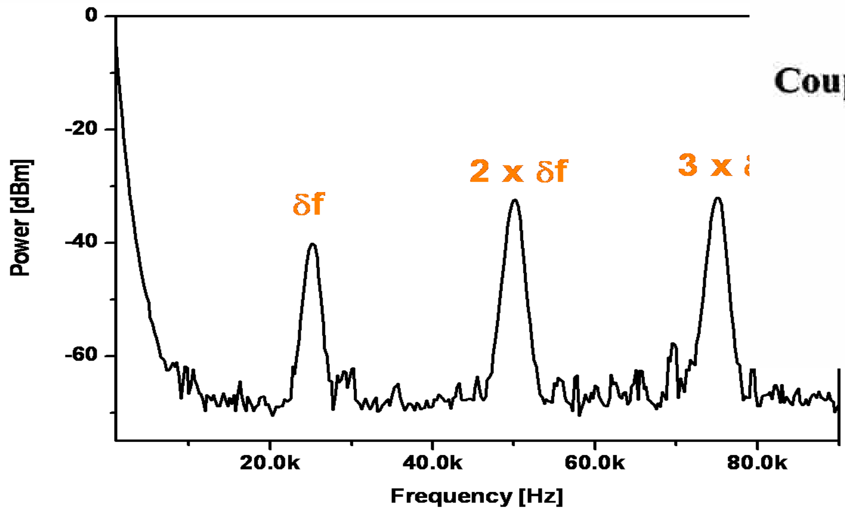


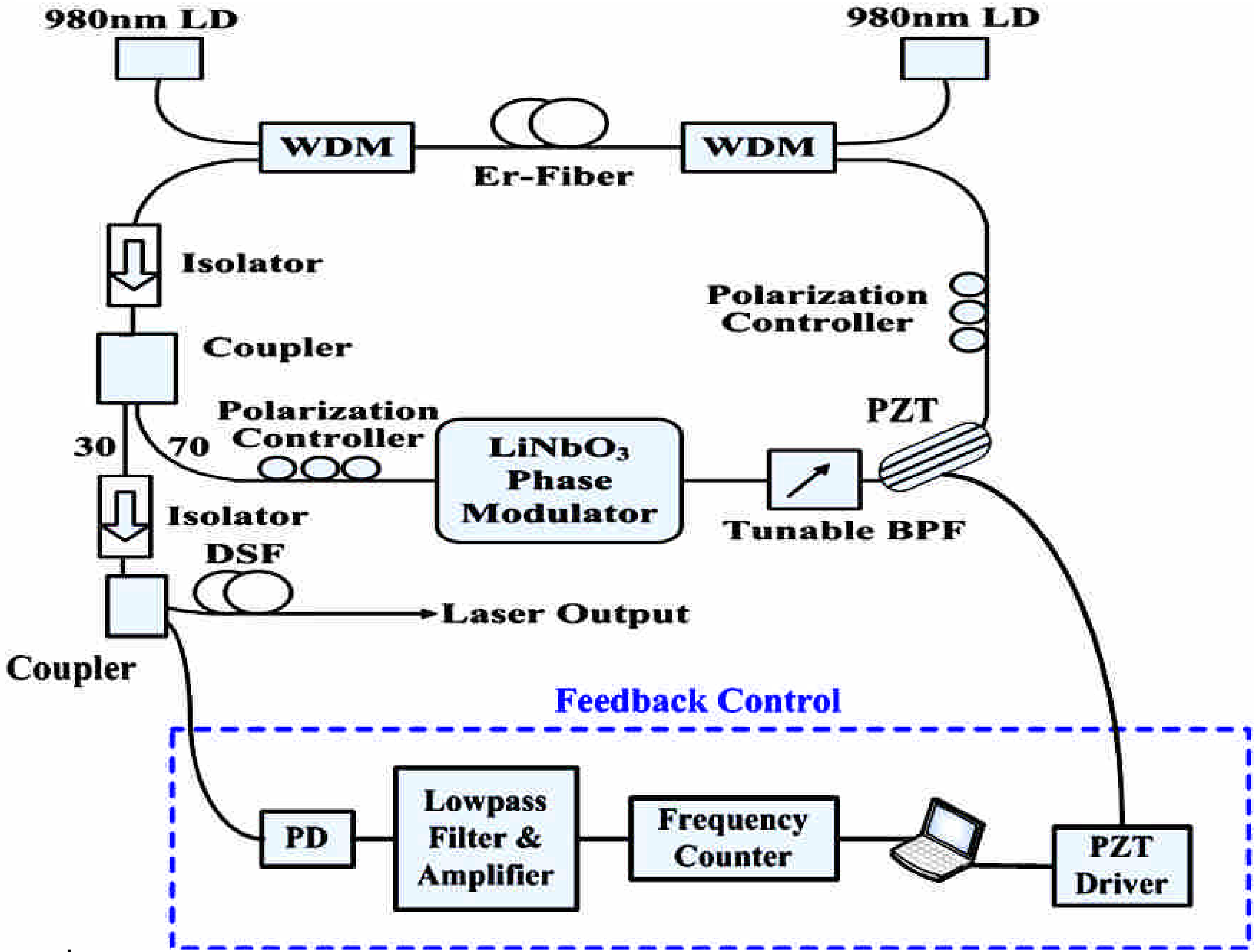
Some interesting observations:

1. **The oscillation of pulse parameters may not be pure sinusoidal.**
2. **The pulse energy may also have small periodic oscillation. (consistent with the near DC measurement results)**

New long term stabilization scheme





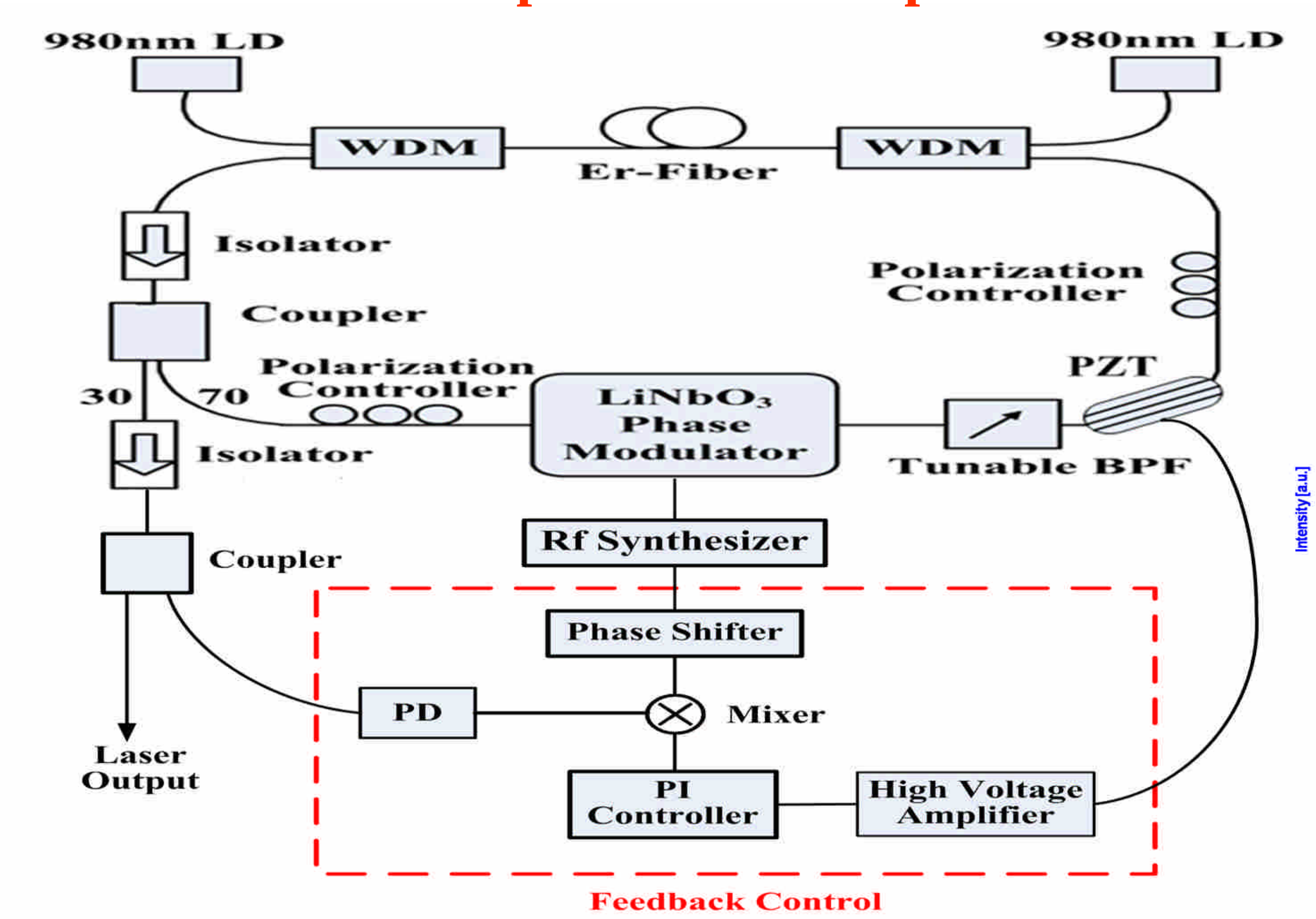


Only low frequency electronics are required for feedback control!

W.-W. Hsiang, C.-Y. Lin, N.-K. Sooi, Y. Lai, Optics Express 14, 1822-1828 (2006).

Bound Soliton Modelocking (I)

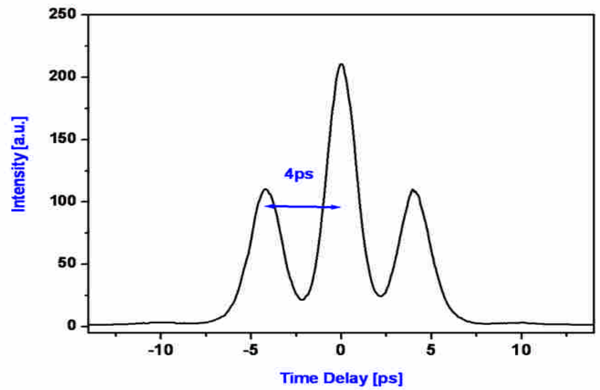
Experimental Setup



W. -W. Hsiang, C. -Y. Lin, and Y. Lai, Opt. Lett. 31, 1627-1629 (2006).

First Observed In 10GHz Hybrid Harmonic Mode-Locking !!

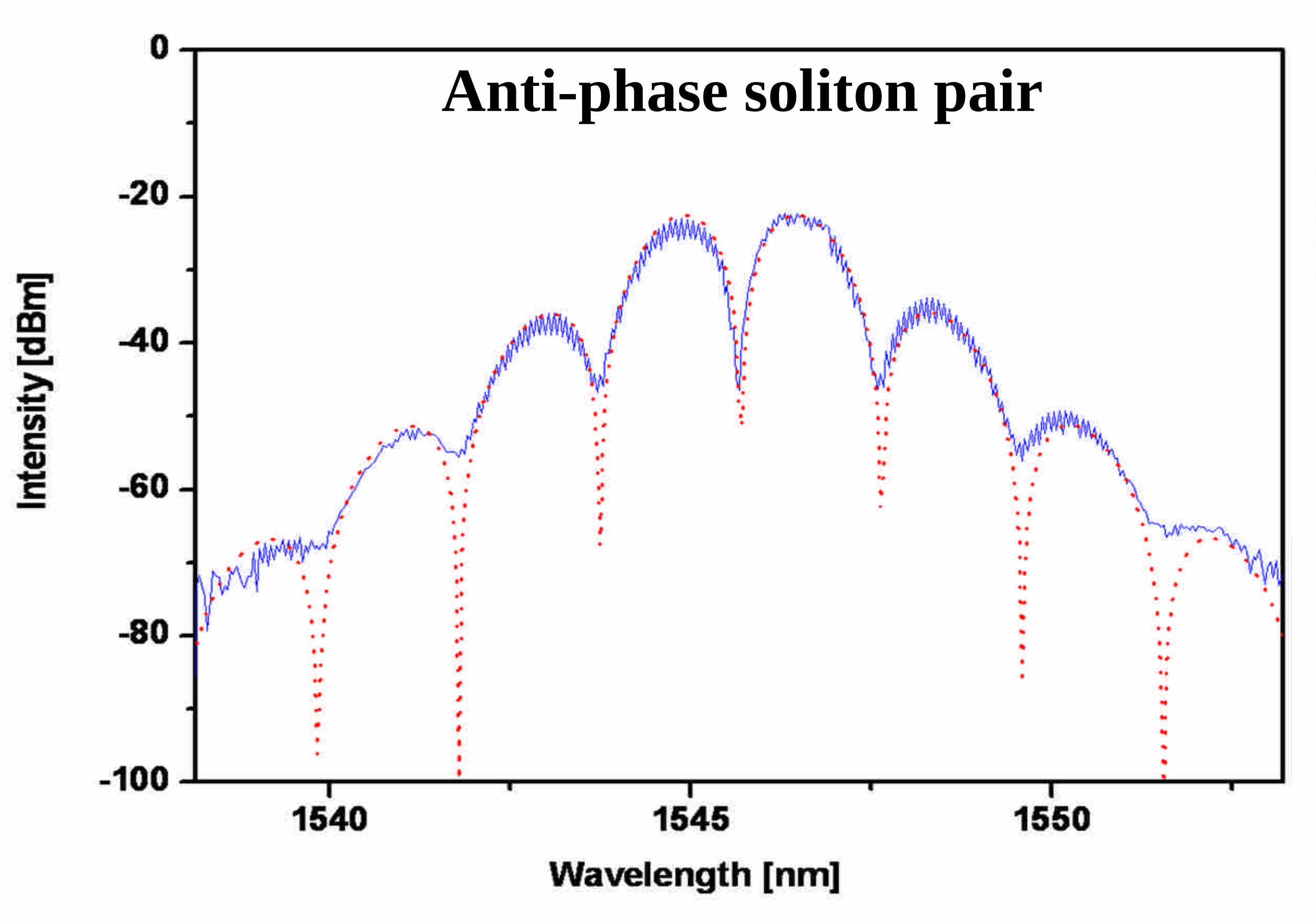
Auto-correlation



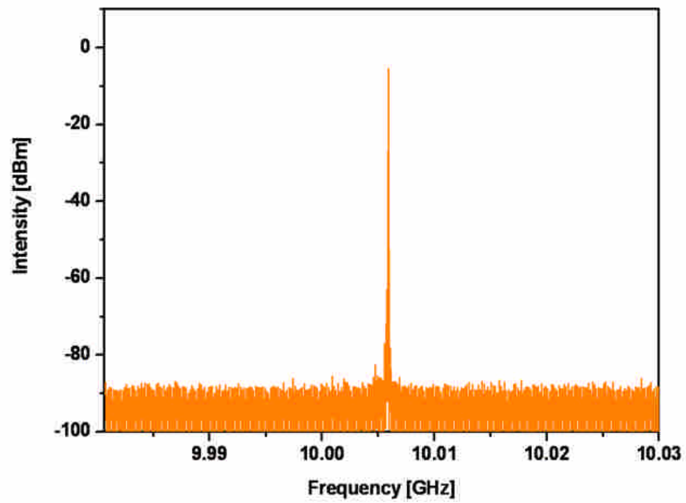
Pulsewidth=1.3ps
Separation=4ps
Equal amplitude soliton pair

Bound Soliton Modelocking (II)

Optical spectrum



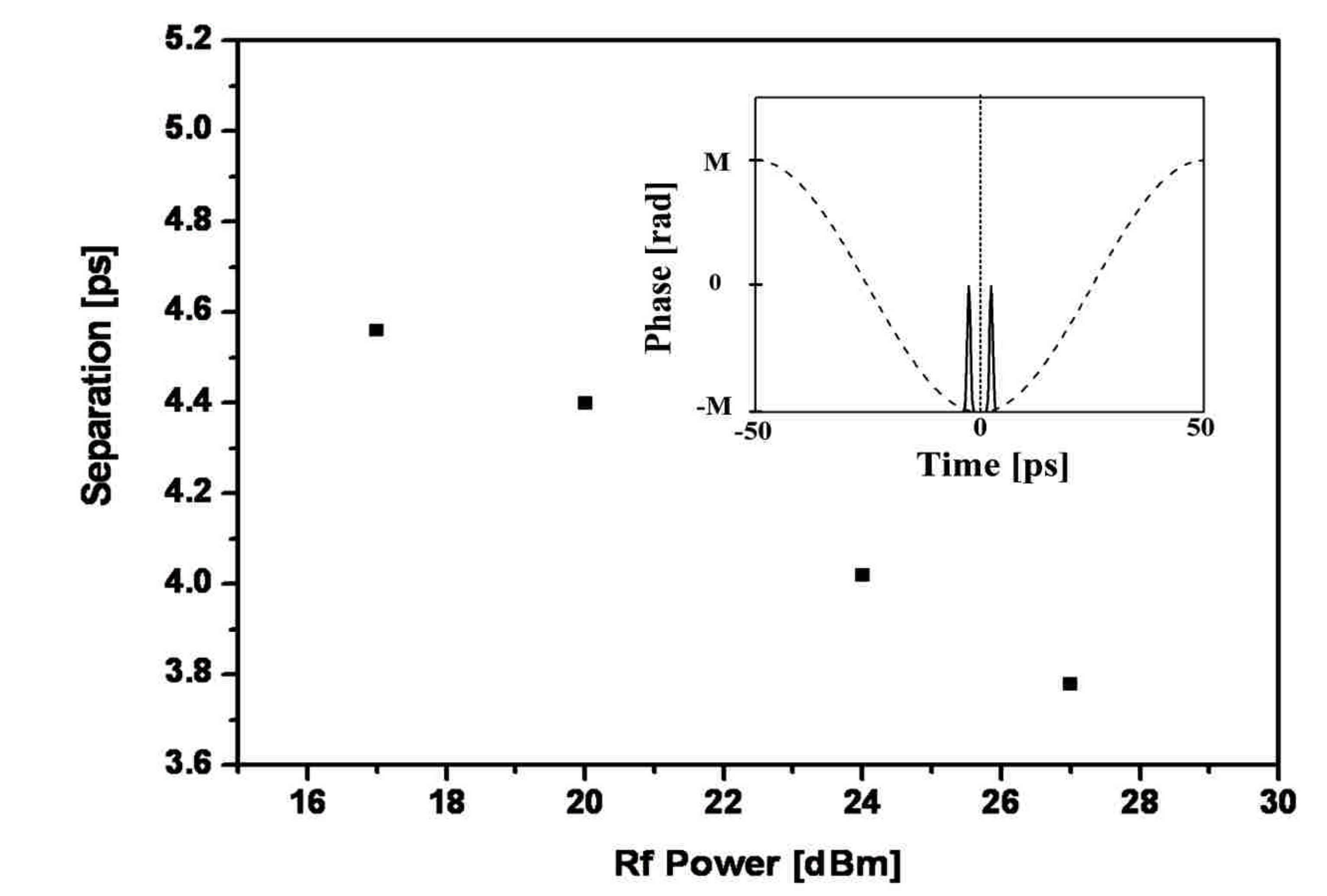
RF spectrum



Very low super-mode Noises!

Bound Soliton Modelocking (III)

Modulation-dependent soliton time separation



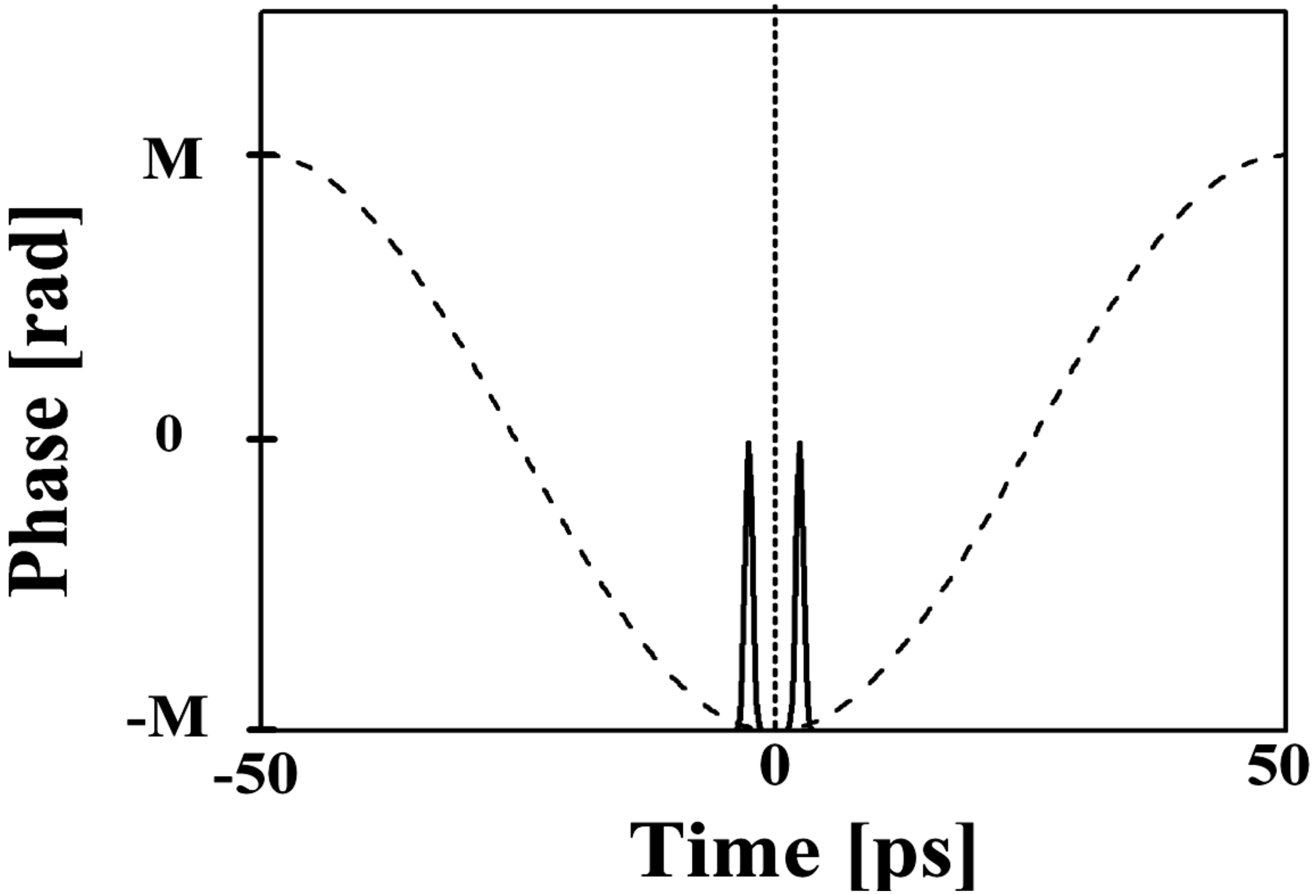
Bound Soliton Modelocking (IV)

Physical mechanism

Phase modulation induced frequency shift

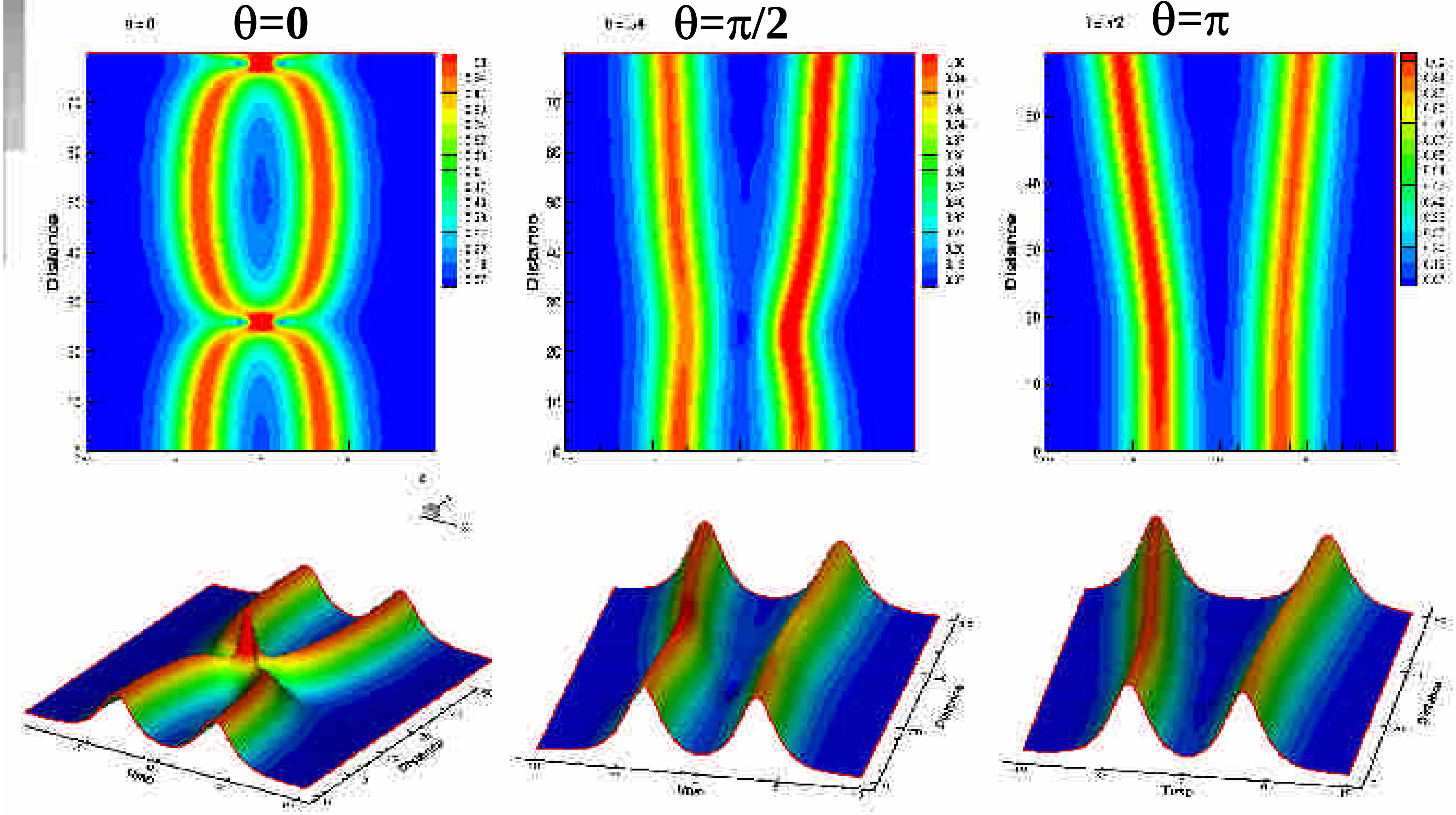
Balanced by

Direct (anti-phase) soliton interaction induced frequency shift



Phase-dependent Soliton interaction

$$U(z,t) = sech(z,t+\rho) + r\,sech(z,t_\rho)e^{i\theta}$$



Laser Dynamics of Bound Soliton Modelocking (I)

$$\frac{\partial U}{\partial T} = \left(\frac{g_0}{1 + \frac{\int_{-\infty}^{\infty} |u(T,t)|^2 dt}{E_S}} - l_0 \right) U + i\, s_m \cos(\omega_m t)\, U + (d_r + i\, d_i) \frac{\partial^2 U}{\partial t^2} + (k_r + i\, k_i)\, U^* U^2$$

$$U(T,t) = a(T) \operatorname{sech}\left[\frac{t - t_0(T)}{\tau(T)}\right] \exp\left[ib(T) \ln\left[\operatorname{sech}\left[\frac{t - t_0(T)}{\tau(T)}\right]\right] + i\omega(T)(t - t_0(T)) + i\theta(T)\right]$$

$$- a(T) \operatorname{sech}\left[\frac{t + t_0(T)}{\tau(T)}\right] \exp\left[ib(T) \ln\left[\operatorname{sech}\left[\frac{t + t_0(T)}{\tau(T)}\right]\right] - i\omega(T)(t + t_0(T)) + i\theta(T)\right]$$

$$\frac{d\omega}{\mathrm{dT}} = -s_m \omega_m \sin[\omega_m t_0(T)]$$

Phase modulation term

$$- \frac{4\, d_r \left(1 + b(T)^2\right) \omega(T)}{3\, \tau(T)^2}$$

Filter damping term

$$+ \frac{1}{\tau(T)} \left(a(T)^2 (-3\, k_i + 3\, k_r\, b(T)) \right) \left(-\frac{2}{3} \right) \mathrm{Exp}[-t_0(T)]$$

Soliton interaction term

Noise Reduction Characteristics

Experimentally, both the asynchronous modelocking and bound soliton modelocking exhibit obvious noise reduction characteristics.

1. Asynchronous modelocking:
   Equivalent sliding filter effects

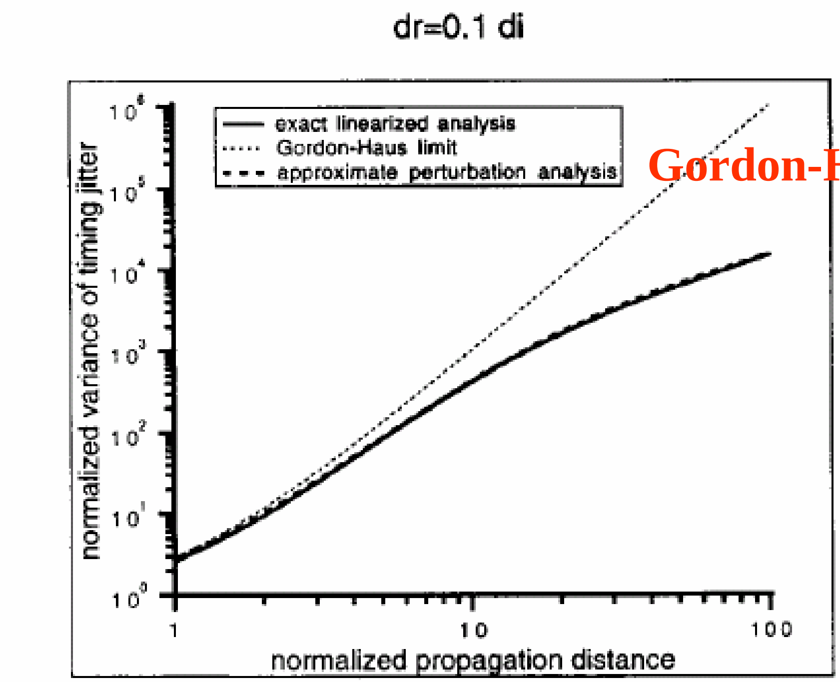
2. Bound soliton modelocking:
   Anti-phase cancellation effects
   Equivalent sliding filter effects

Solitons transmission control with optical filtering

Linear frequency fluctuations => Cubic timing jitter (Gordon-Haus effects)

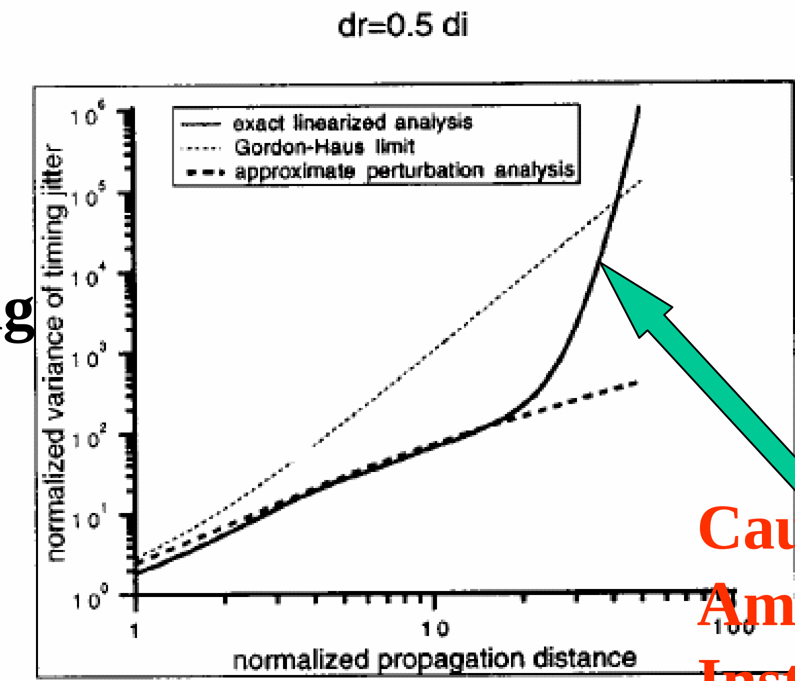
Optical filtering => limited frequency fluctuations => linear timing jitter

Sliding Optical filtering => Removal of amplitude instability



Gordon-Haus limit

Timing Jitter



Can be Removed By Sliding Filters

Caused by Amplitude Instability

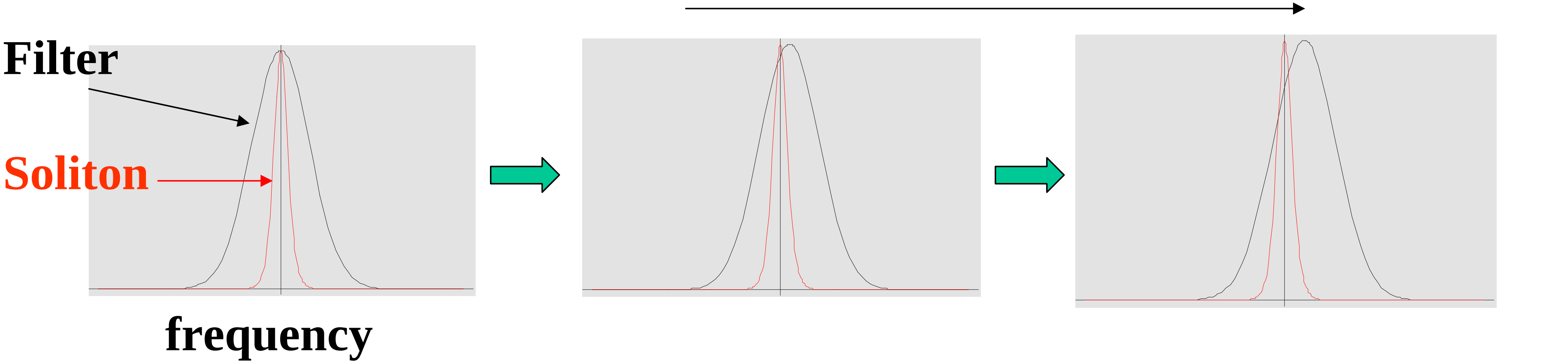
1. A. Mecozzi and J. D. Moores, H. A. Haus, and Y. Lai, Optics Letters, vol.16, 1841, 1991.
2. Y. Lai, IEEE J. Lightwave Tech. 11. 462, 1993.

Soliton perturbation theory

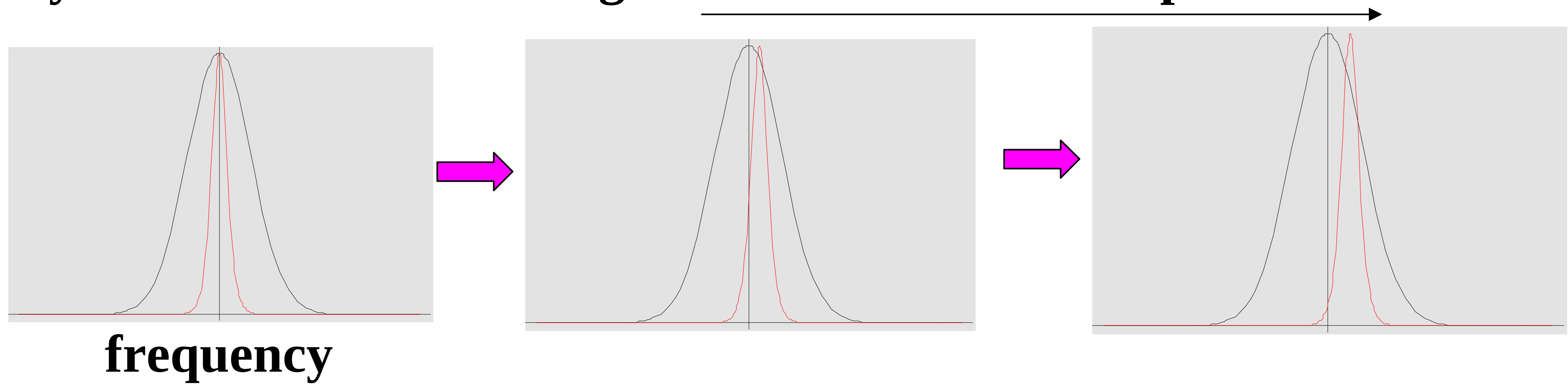
ideal soliton	amplifier+loss	amplifier+loss+filter
$\frac{d\Delta p}{dz} = 0$	$\frac{d\Delta p}{dz} = n_p(z)$	$\frac{d\Delta p}{dz} = -\gamma_p \Delta p + n_p(z)$
$\frac{d\Delta t_0}{dz} = \Delta p$	$\frac{d\Delta t_0}{dz} = \Delta p + n_t(z)$	$\frac{d\Delta t_0}{dz} = \Delta p + n_t(z)$
	$Var[\Delta t_0(z)] \propto z^3$	$Var[\Delta t_0(z)] \propto z$
	Gordon-Haus limit	

Asynchronous Modelocking vs Sliding Filter

Sliding Filter:



Asynchronous Modelocking:



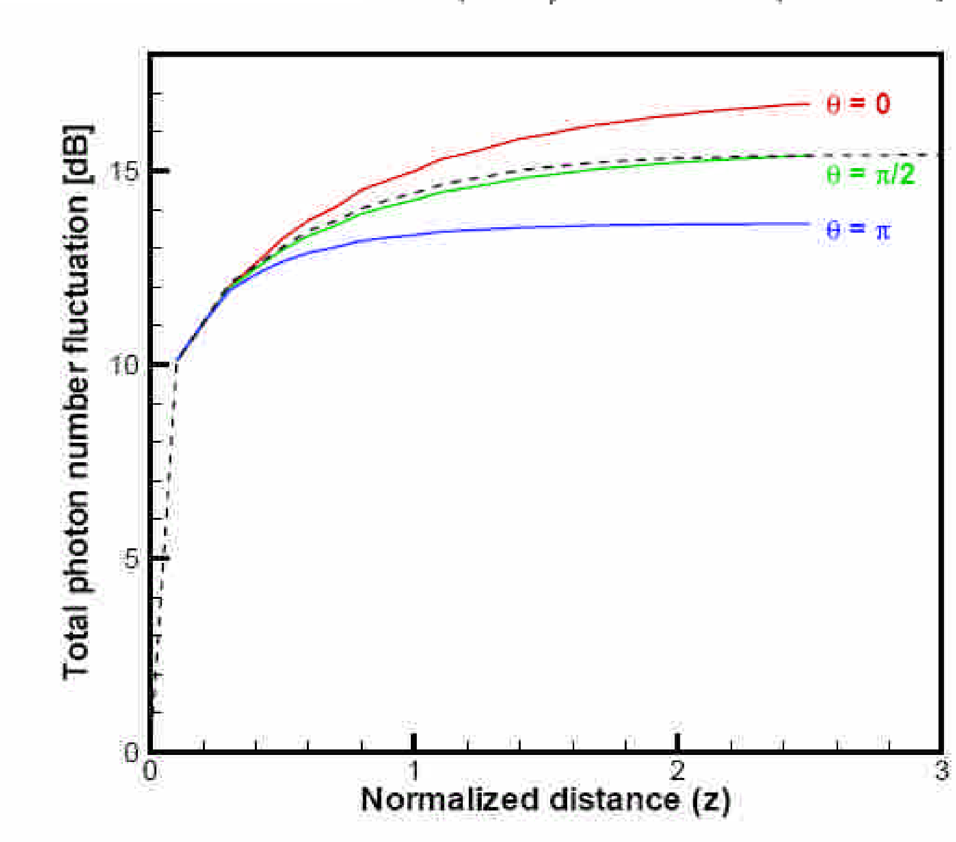
Solitons can survive under the sliding, whereas CW noises see larger loss.

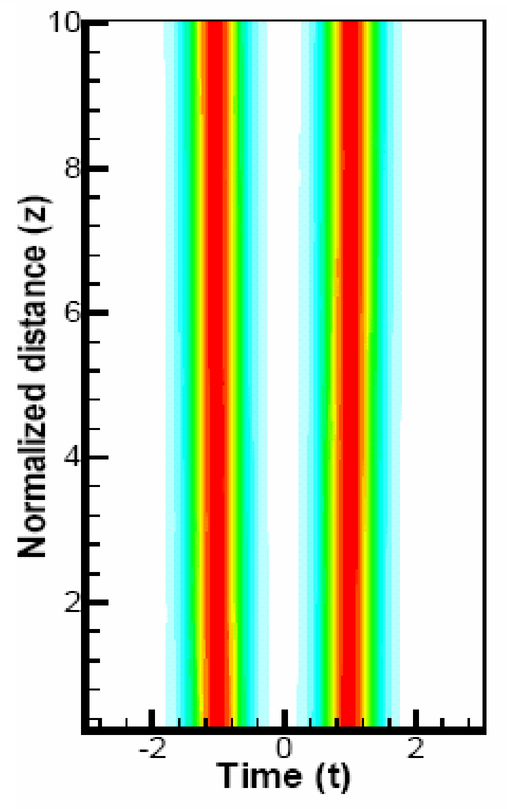
Bound solitons in passive modelocked fiber lasers : theory

Anti-phase noise cancellation

$$iU_z + (D/2)U_{tt} + |U|^2U = i\delta U + i\epsilon|U|^2U + i\beta U_{tt} + i\mu|U|^4U - \nu|U|^4U,$$

$$U(z,t) = sech(z, t+\rho) + r\, sech(z, t_\rho)e^{i\theta}$$

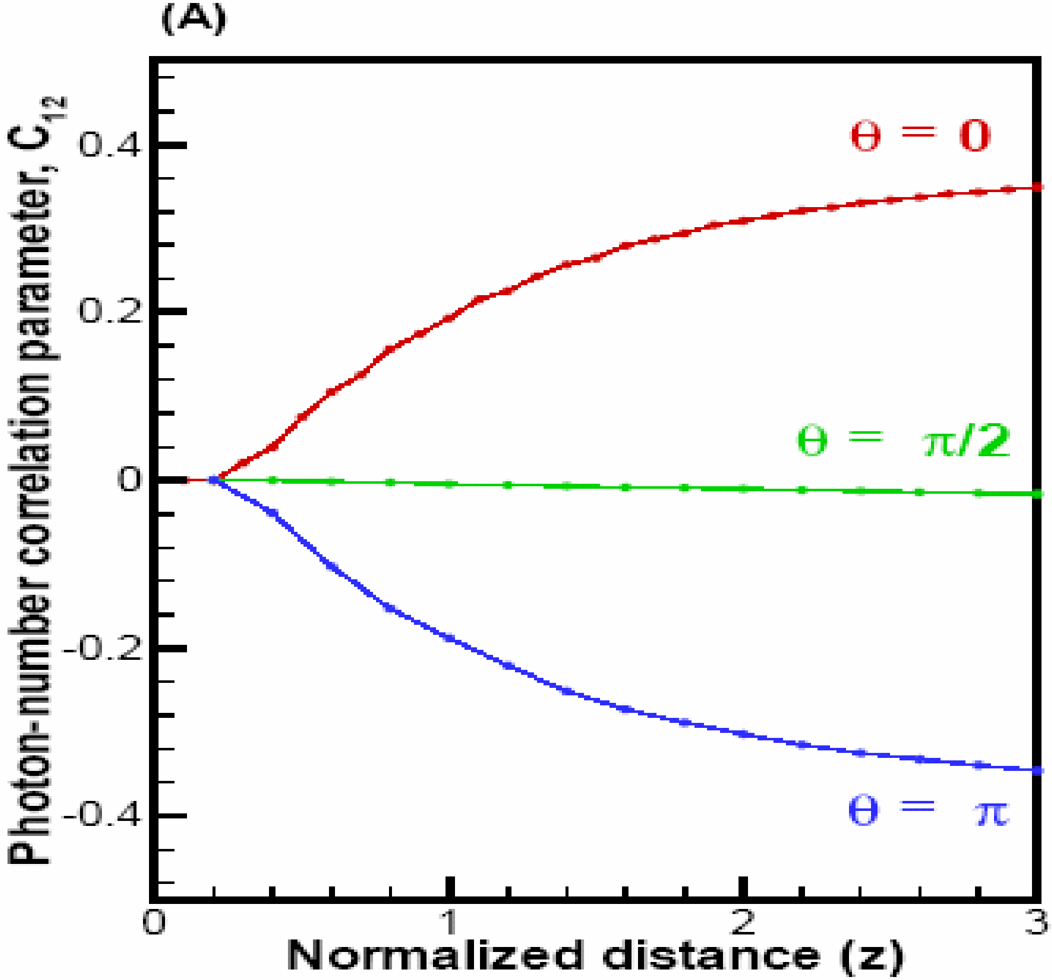




R.-K. Lee, Y. Lai, and B. A. Malomed, Opt. Lett. 30, 3084-3086 (2005).

Quantum Correlation?

$$U(z,t) = sech(z, t+\rho) + r\, sech(z, t_\rho)e^{i\theta}$$



$$C = \frac{\left\langle : \Delta\hat{A}\Delta\hat{B} : \right\rangle}{\sqrt{\left\langle \Delta\hat{A}^2 \right\rangle \left\langle \Delta\hat{B}^2 \right\rangle}}$$

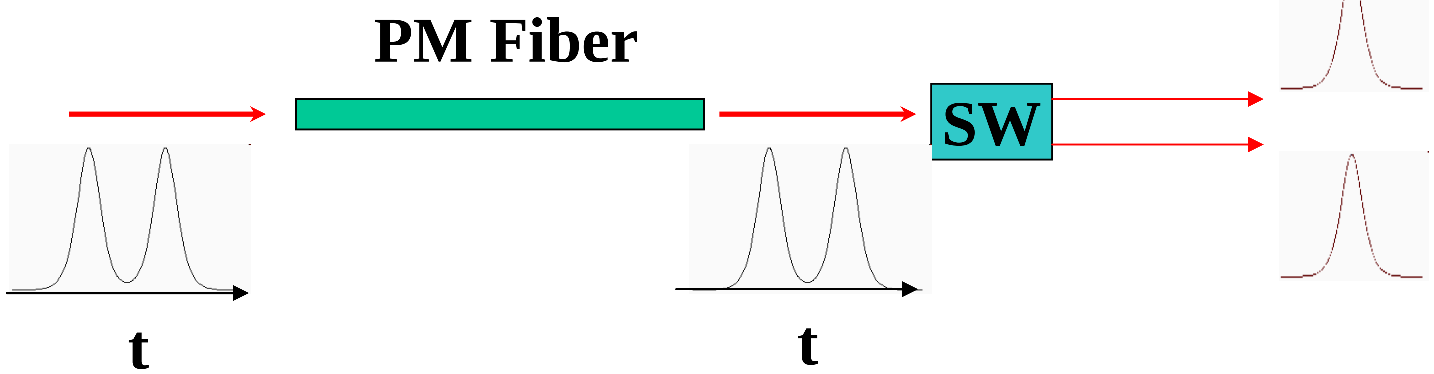
$0 < C \leqq 1$: Positive correlation

$C = 0$: No correlation

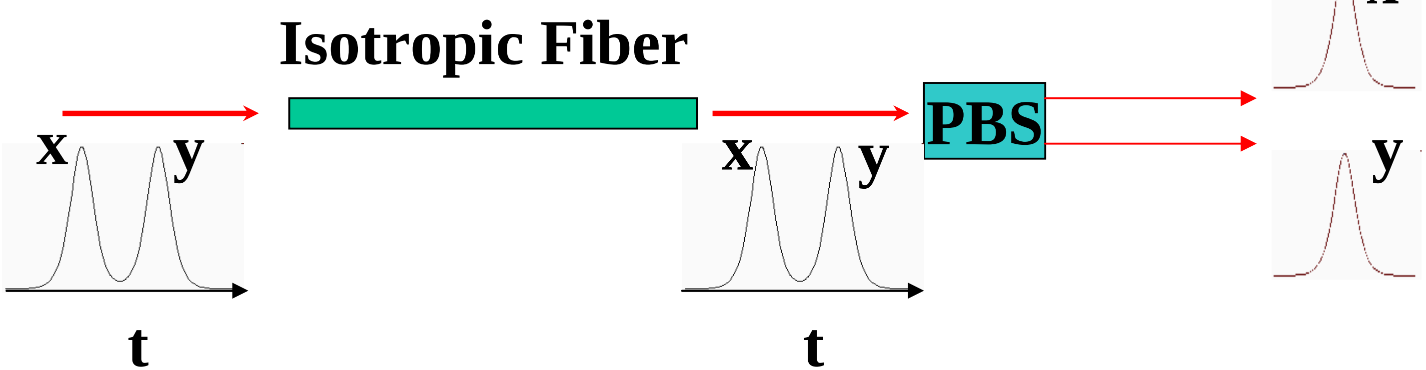
$-1 \leqq C < 0$: Negative correlation

Quantum Correlated Soliton Pairs

(1) TDM soliton pair



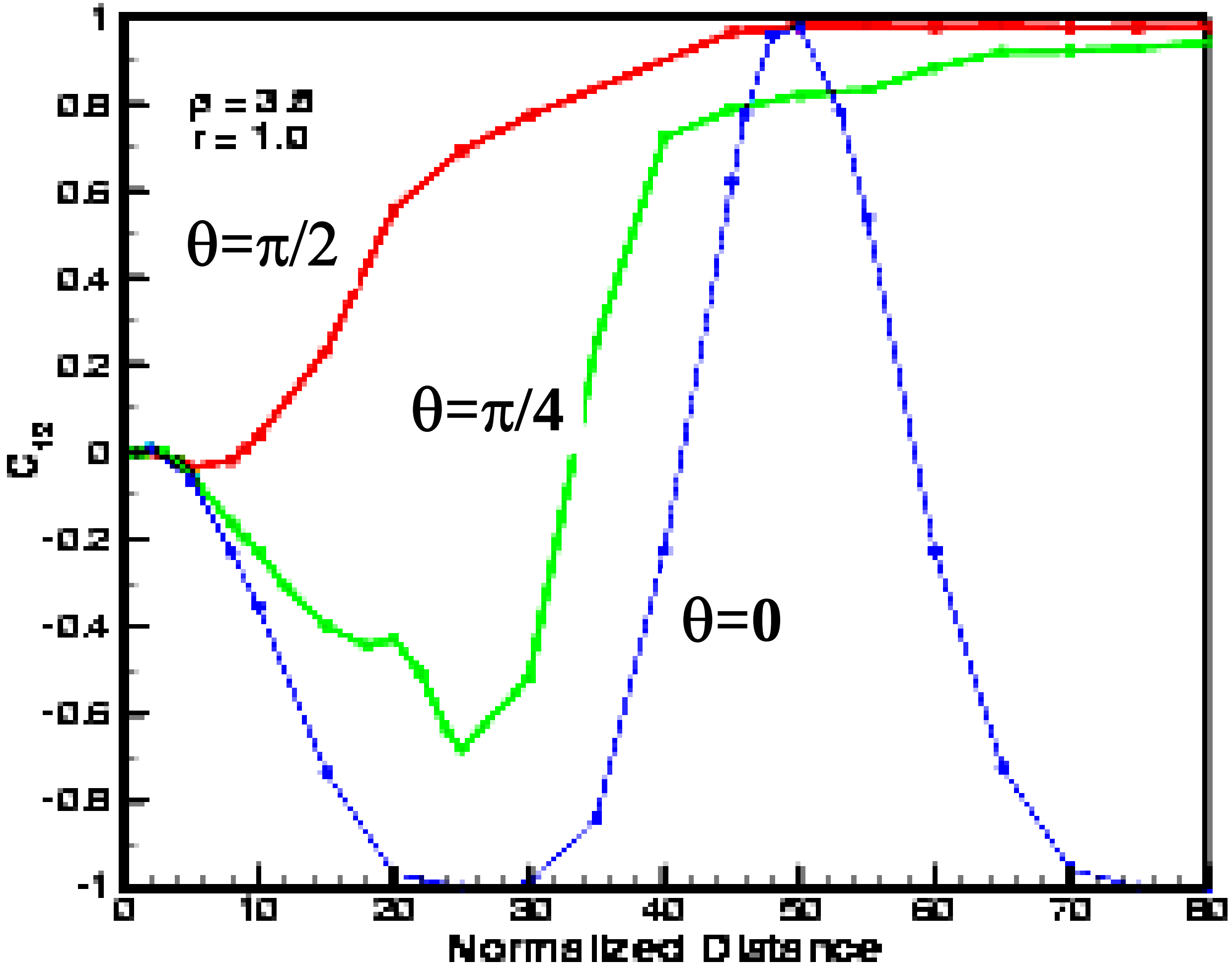
(2) PDM soliton pair



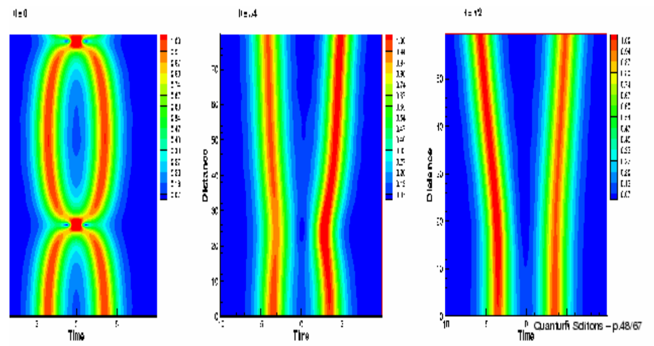
If necessary, the Sagnac loop configuration also can be used.

R.-K. Lee, Y. Lai, and B. A. Malomed, Phys. Rev. A 71, 013816 (2005)

Photon number correlated TDM soliton pair



$$C_{1,2} = \frac{\langle :\Delta\hat{n}_1\Delta\hat{n}_2:\rangle}{\sqrt{\Delta\hat{n}_1^2}\sqrt{\Delta\hat{n}_2^2}}$$



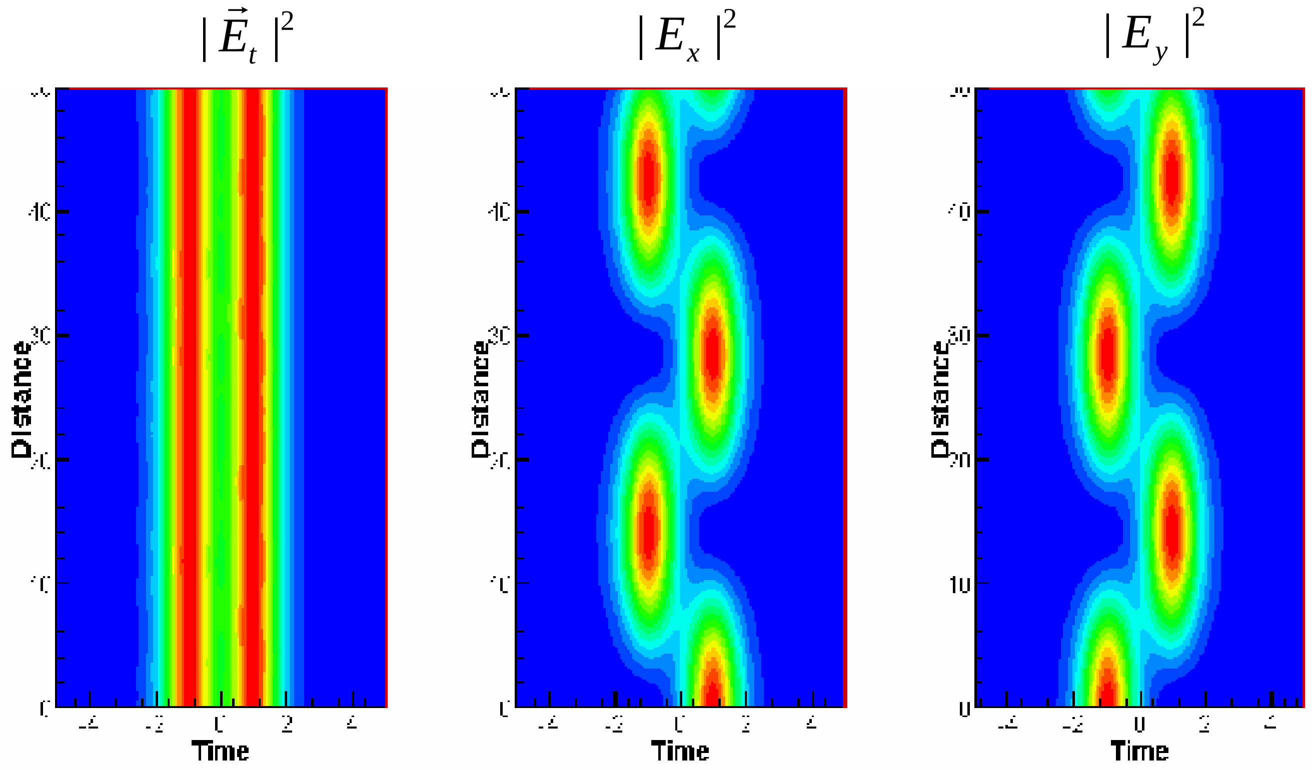
R.-K. Lee , Y. Lai, and B. A. Malomed, Phys. Rev. A 71, 013816 (2005)

CNLSE PDM bound solitons

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\frac{\partial^2 U}{\partial t^2} + A|U|^2 U + B|V|^2 U = 0$$
$$i\frac{\partial V}{\partial z} + \frac{1}{2}\frac{\partial^2 V}{\partial t^2} + A|V|^2 V + B|U|^2 V = 0$$

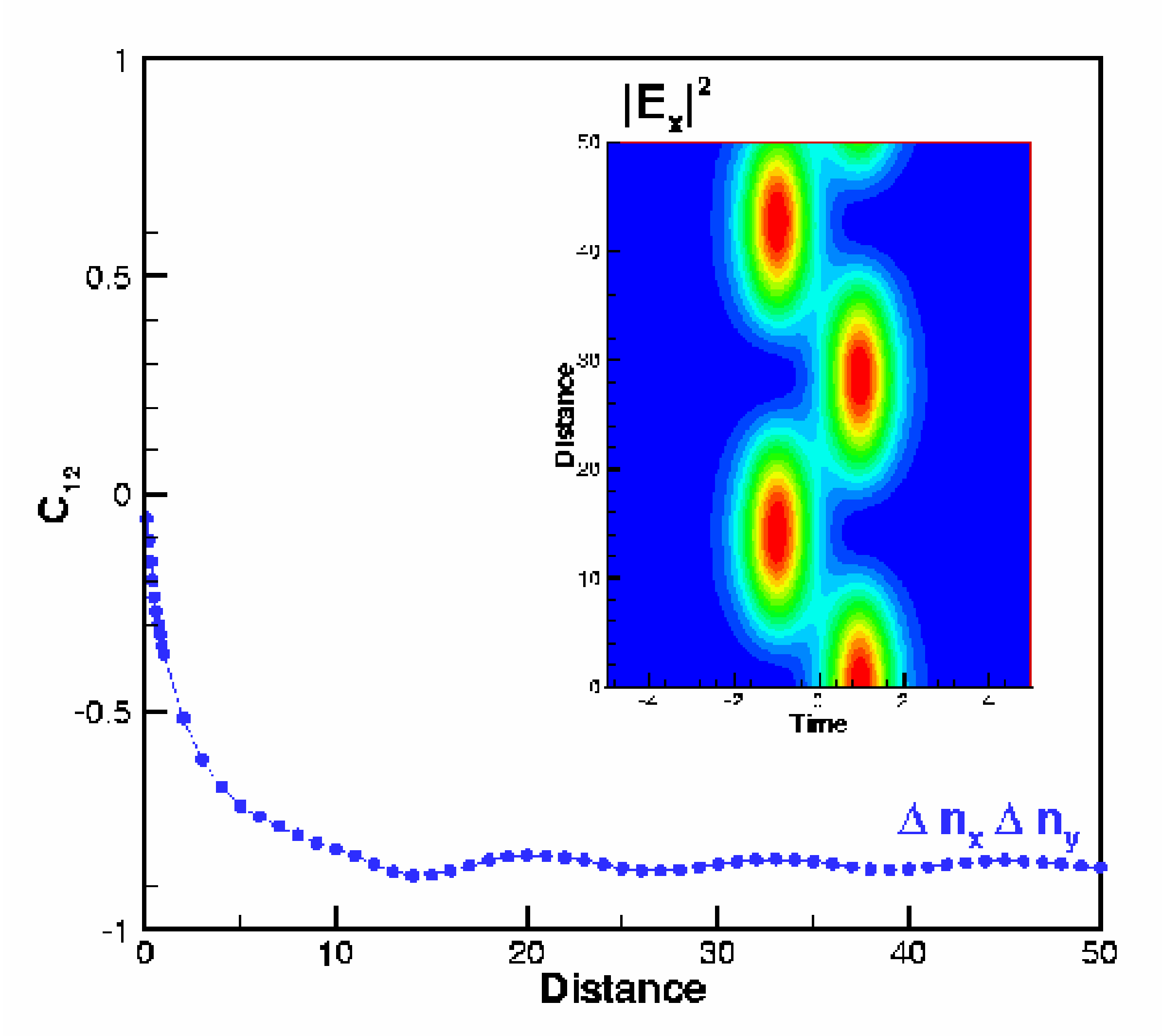
U,*V*: Fields in circular polarizations

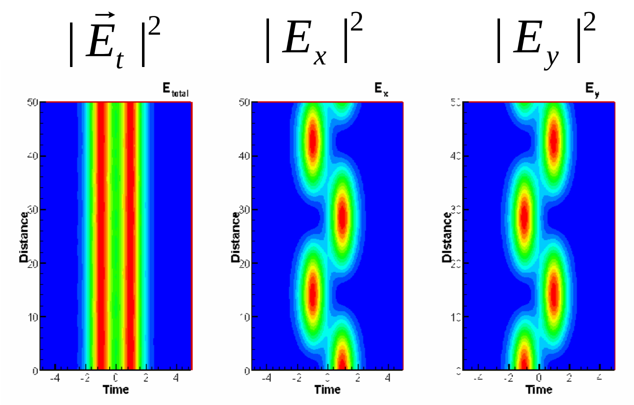
and $A = 1/3$, $B = 2/3$



M. Haelterman et al., Optics Letters 18, 1406 (1993).

Quantum correlation of PDM soliton pairs



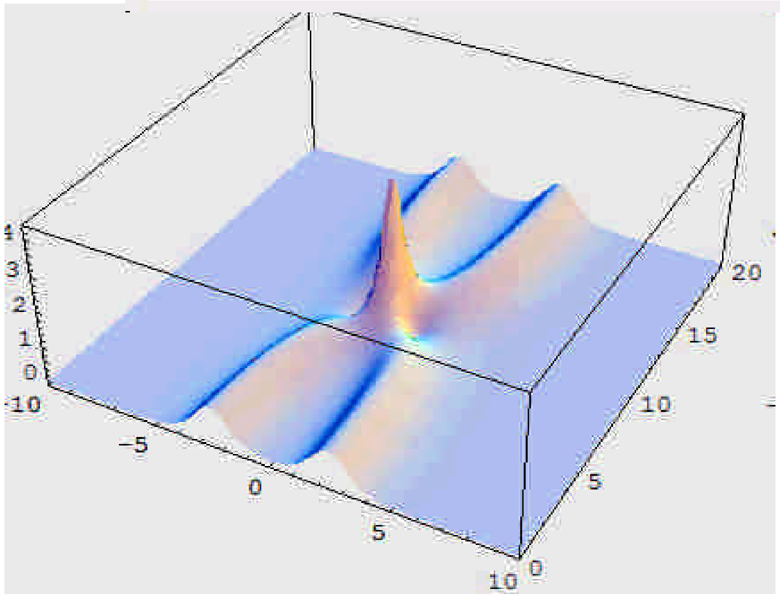


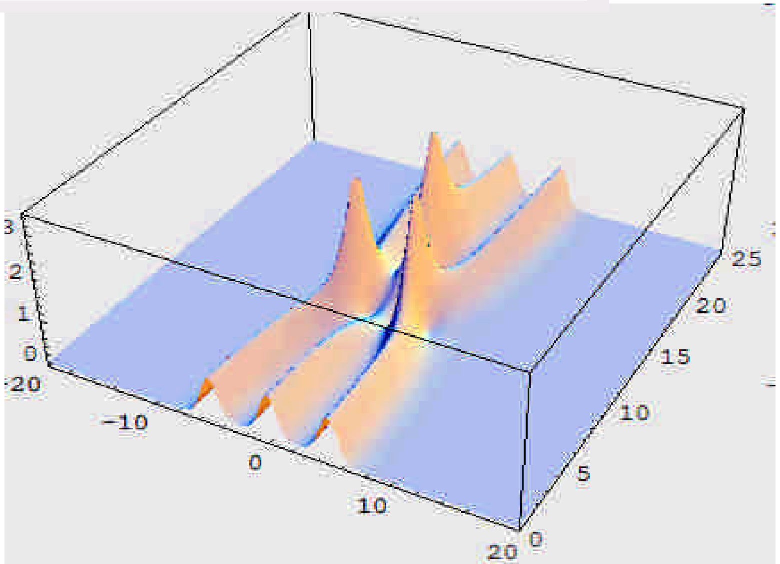
R.-K. Lee, Y. Lai, and B. A. Malomed, Phys. Rev. A 71, 013816 (2005)

Proof of Quantum Entanglement

We rigorously prove that the time-multiplexed optical solitons become quantum entangled in the sense that their "quadrature components of internal modes" satisfy the EPR non-local criterion: the uncertainty product of the inferred quadrature components is below the Heisenberg uncertainty product limit.

$$\text{Squeezing ratio of } \mathrm{Var}\left[\hat{q}_1+\hat{q}_2\right]\mathrm{Var}\left[\hat{p}_1-\hat{p}_2\right] \le \frac{\lambda_{\mathrm{opt}}}{\lambda_{\mathrm{snd}}} < 1$$





To be presented at CLEO2008

Possible Applications

1. Optical communication
   (1) economic stable optical pulse sources
   (2) dispersion measurement
   (3) using bound solitons for noise cancellation?
2. Optical sampling: automatic periodic timing position scan using ASM fiber lasers
3. Tunable THz generation using bound solitons
4. Quantum information
   (1) any advantage of using entangled bound solitons in quantum key distribution?

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Conclusions

- Modelocked fiber lasers have become a platform for new soliton phenomena.
- Asynchronous modeloking helps to build stable modelocked fiber soliton lasers.
- A new type of soliton bound pairs in a hybrid modelocked fiber laser is discovered.
- Noise reduction and quantum entanglement of soliton pairs are predicted.
- Possible applications need more investigation.