Transport Capacity and Spectral Efficiency of Large Wireless CDMA Ad Hoc Networks

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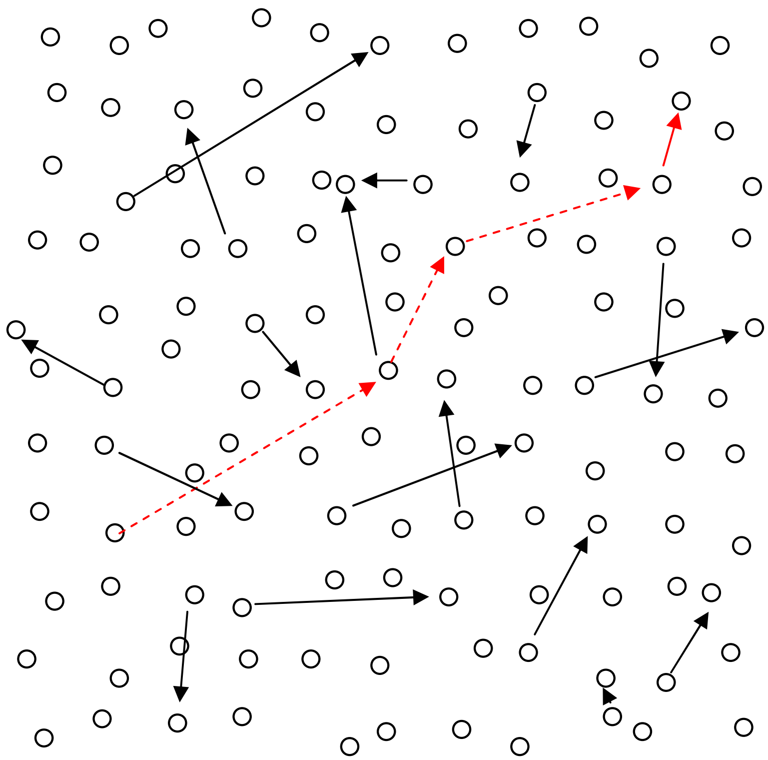
Wireless Ad Hoc Network

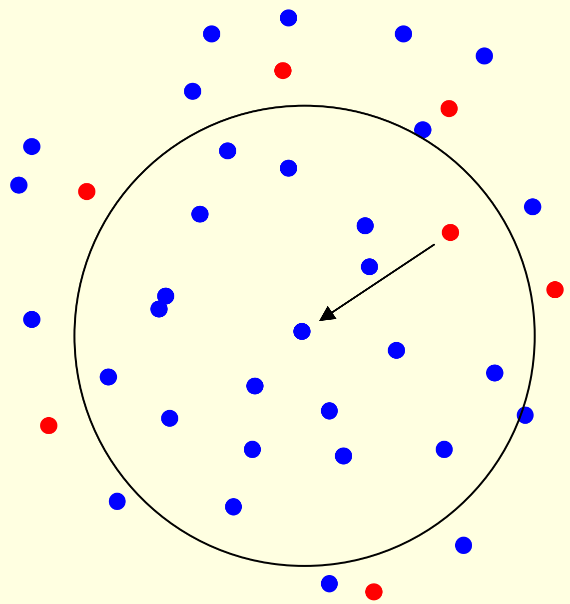


A Fundamental Question

- What is the information-theoretical limit
  - Transport capacity (packet-meters/slot/node)
  - Spectral efficiency (bit-meters/Hz/second/m^2)

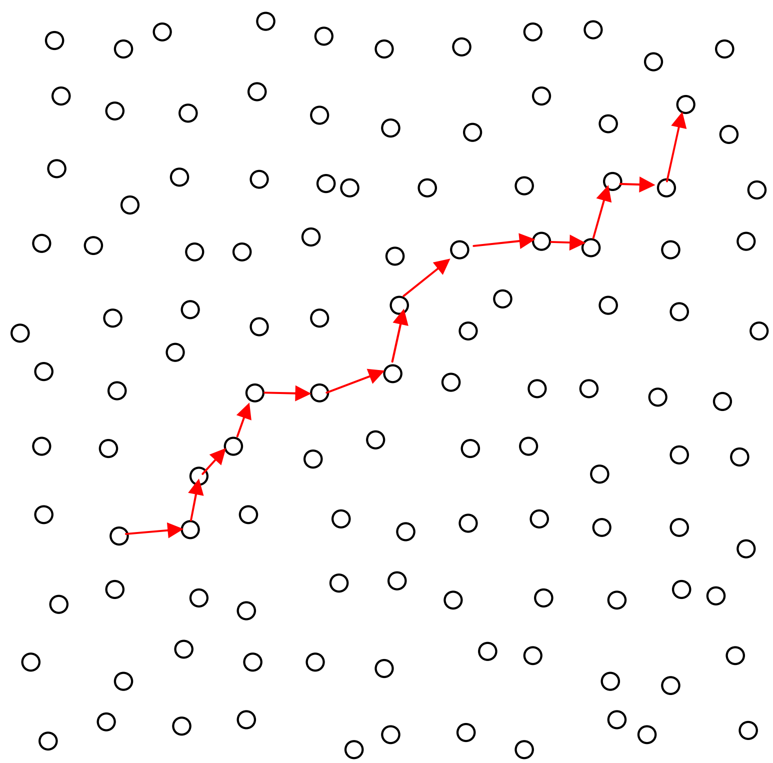
Gupta-Kumar Model (2000)

- Assumption
  - Achievable rate on each link is fixed
  - Effective communications are confined to nearest neighbors



Gupta-Kumar Model (2000)

- For an ad hoc network on a unit square, if node density is *D*, the number of nodes on a path equals about $D^{1/2}$



Gupta-Kumar Scaling Law (2000)

- Scaling law
  - As node density $D \to \infty$, transport capacity converges to zero at rate $O(1/D^{1/2})$

- Large scale wireless ad hoc networks are incapable of information transportation

  – a pessimistic conclusion

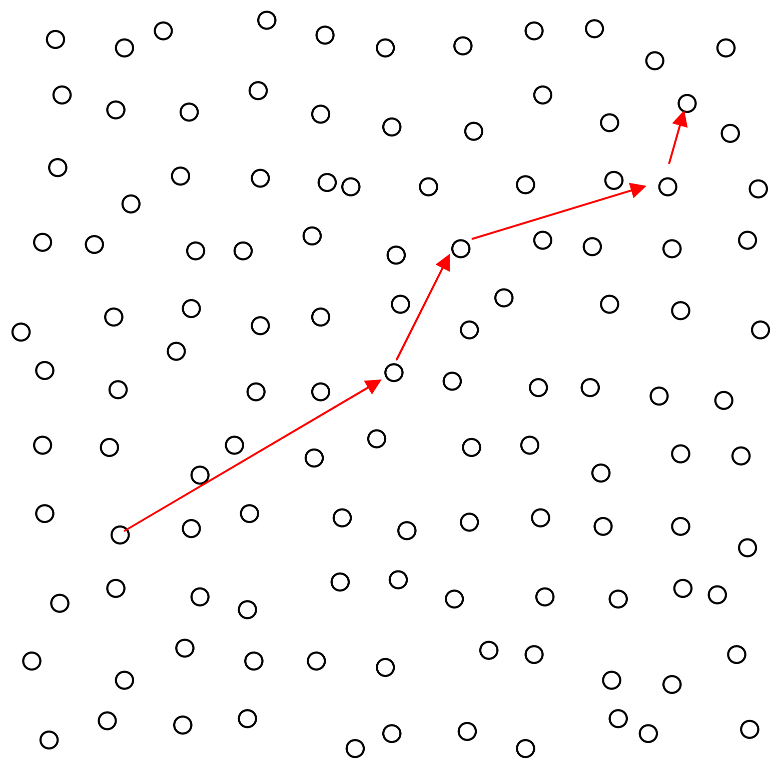
Can Scaling Law be Overcome?

Gupta-Kumar Model

- Communications are confined in nearest neighbors
- Radio frequency bandwidth is not considered in the model
- Spectral efficiency is unknown

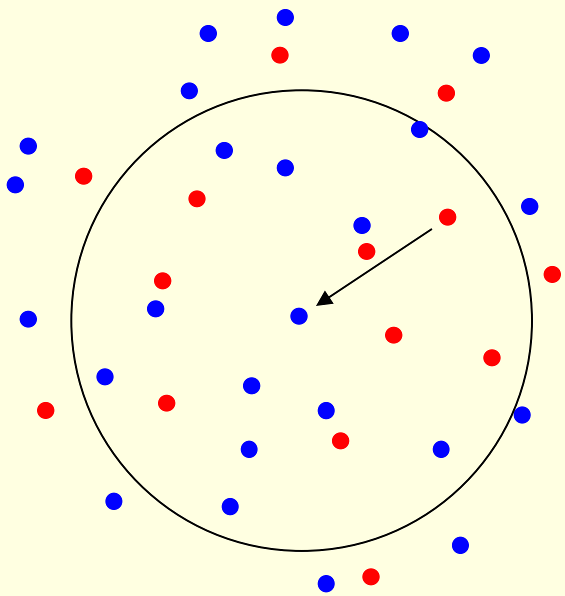
Observation I

- If communications are not confined to nearest neighbors, transport capacity can be increased



Observation II

- If CDMA channel is considered and spreading gain (or bandwidth) is large compared with node density, then communications are not necessary to be confined in nearest neighbors



A wireless CDMA ad hoc network may overcome the scaling law

Our Model

Large Wireless CDMA Ad Hoc Networks

CDMA

- Nodes access each other through a common CDMA channel
- Spreading sequences are random, i.i.d. (long sequences)
- Spreading gain $N = WT_b$
- All nodes have same transmission power P_0
- No power control is employed

Power Decay Model

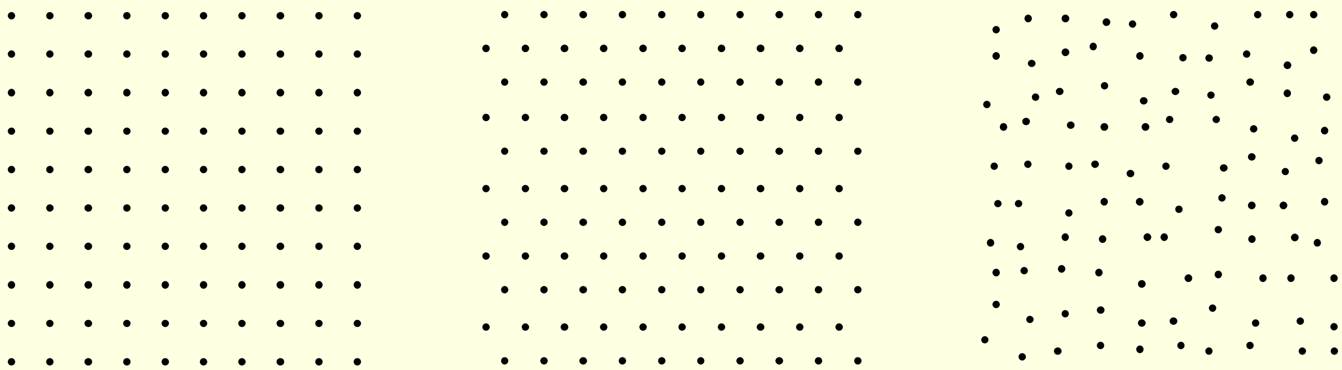
- Power decays in distance *r*

$$P(r) = \frac{P_0}{(r / r_0 + 1)^{\beta}}$$

  - P_0 is transmission power, $r_0 > 0$, $\beta > 2$

Network Topology

- Nodes are distributed on entire 2-D plane
- Node locations can be regular or arbitrary



Node Distributions

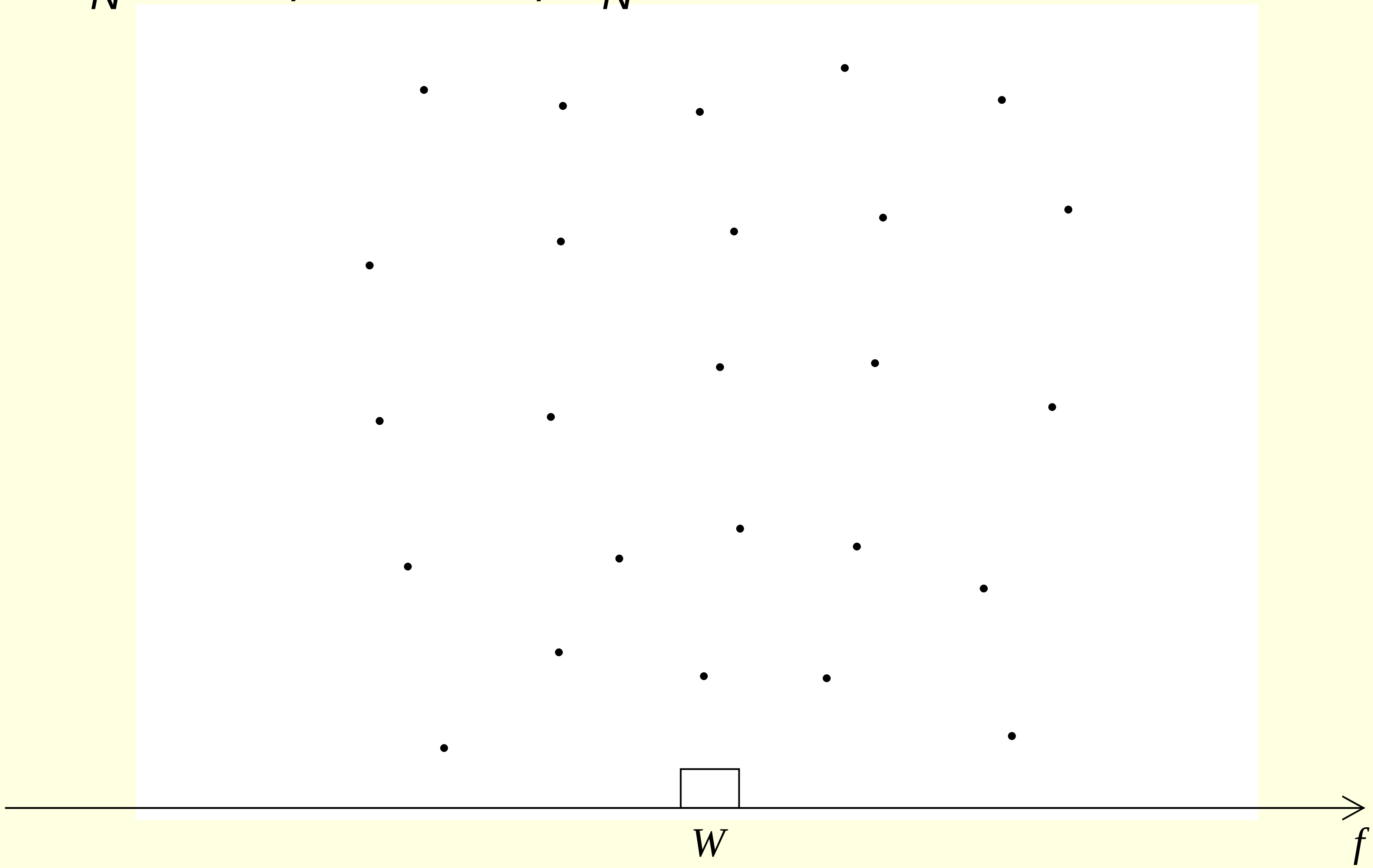
- Nodes are uniformly distributed
- At any time t, a percentage ρ of nodes are sending
- Sending nodes are also uniformly distributed
- For each N, node density is d_N, or

  d_N/N (nodes/Hz/second/m^2)
- Traffic intensity

  $\rho d_N/N$ (sending nodes/Hz/second/m^2)

Limiting Network

- $d_N \to \infty$, $N \to \infty$, $d_N/N \to \alpha$



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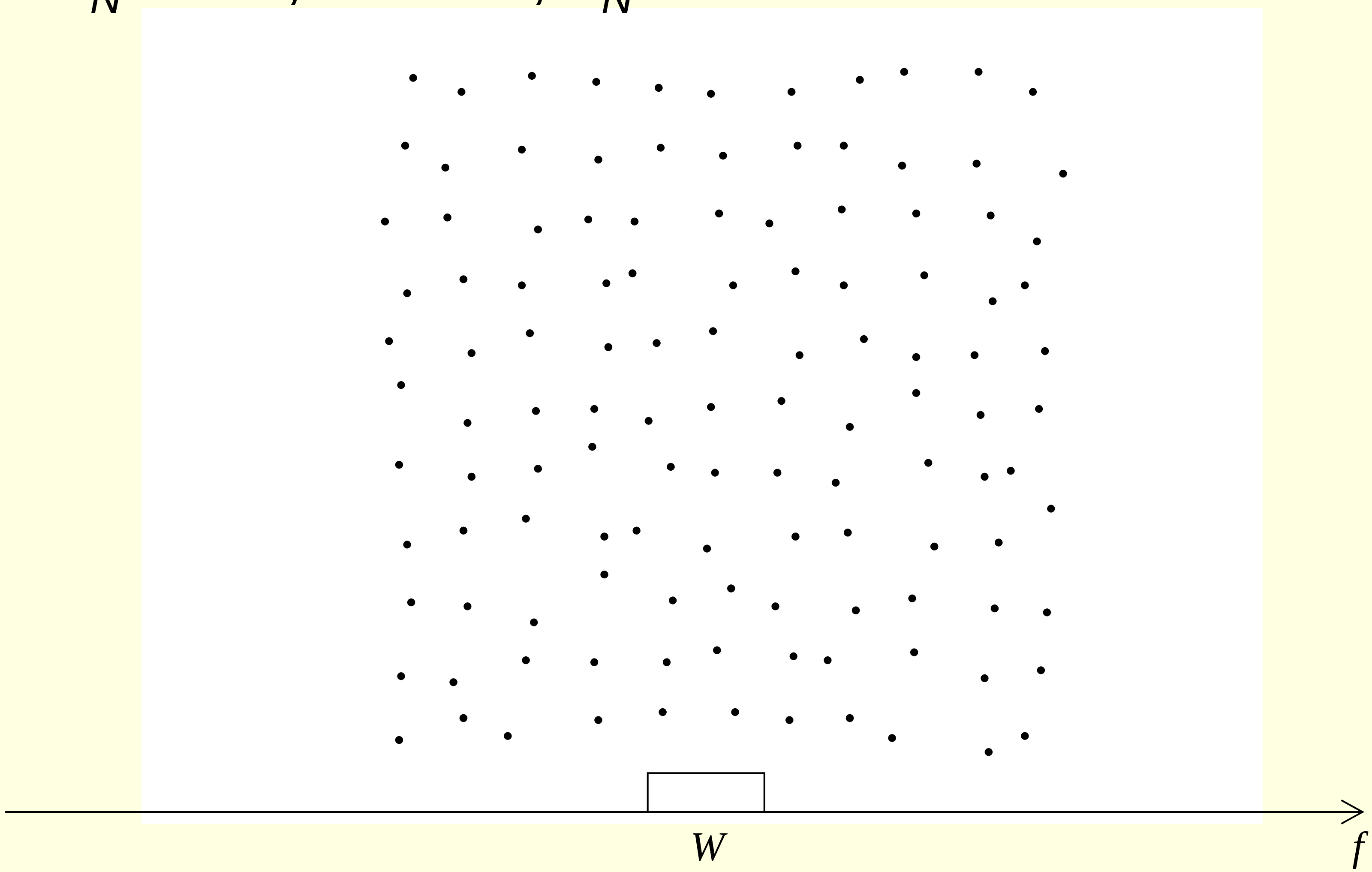

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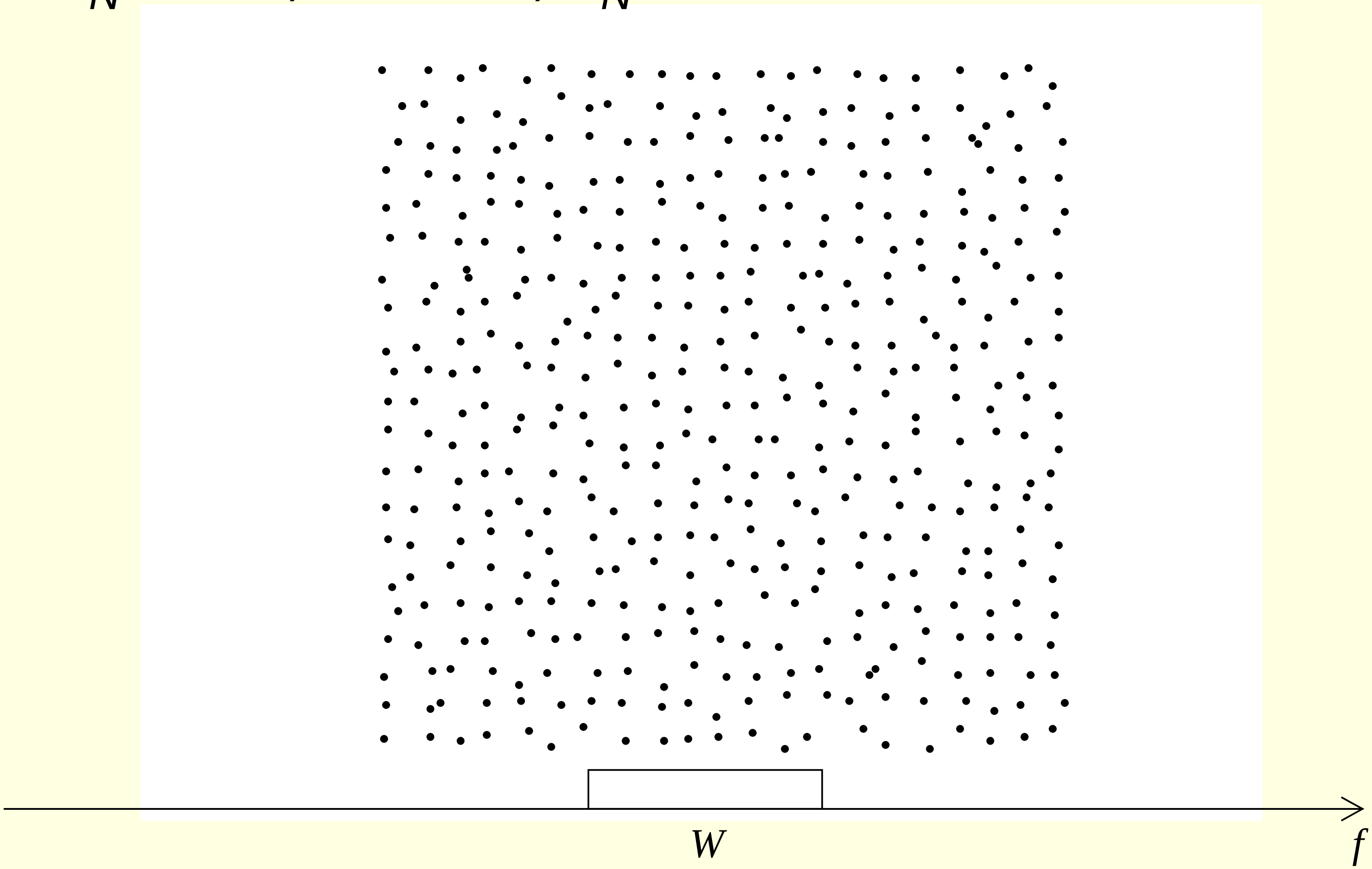

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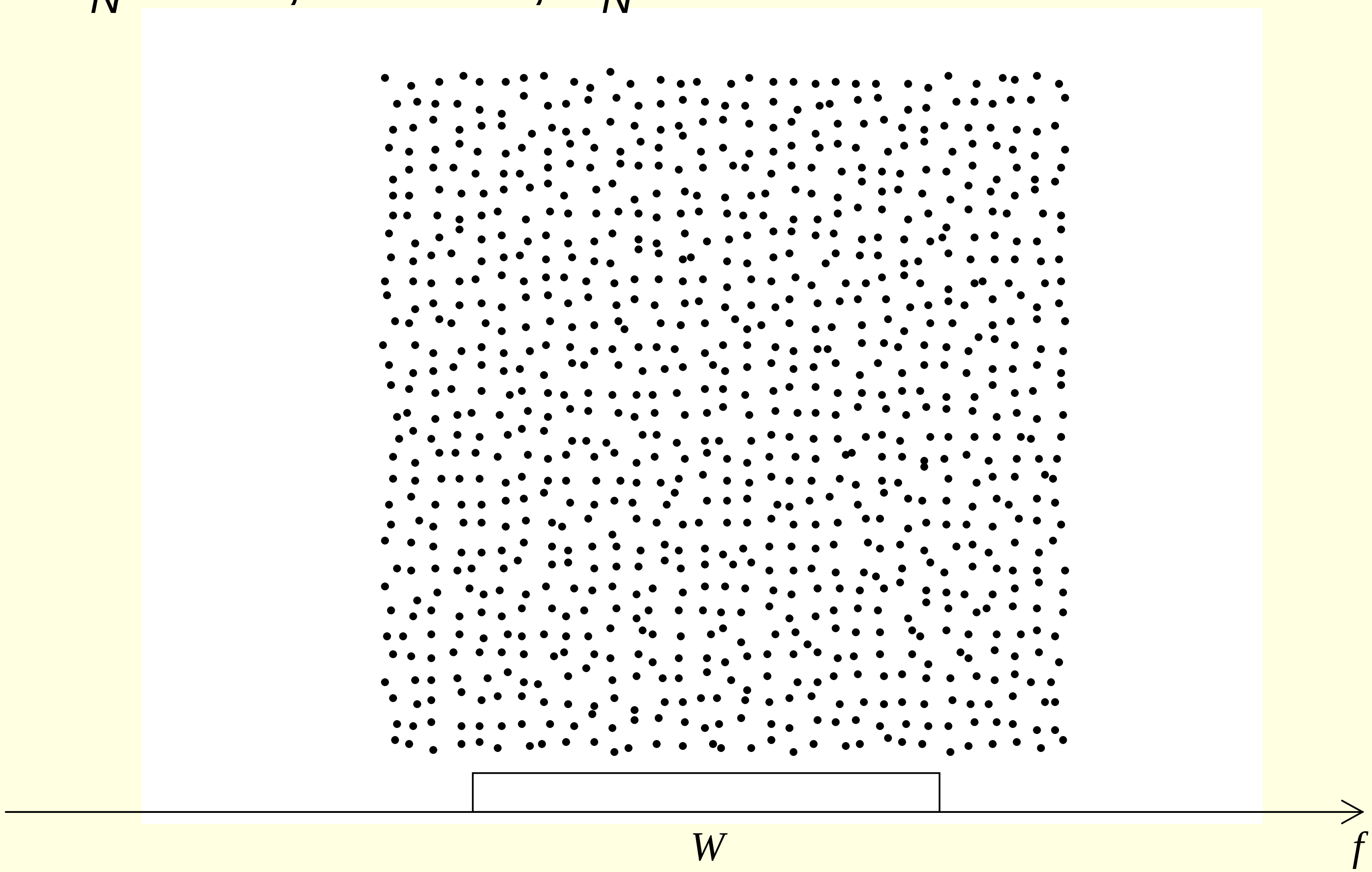

Limiting Network

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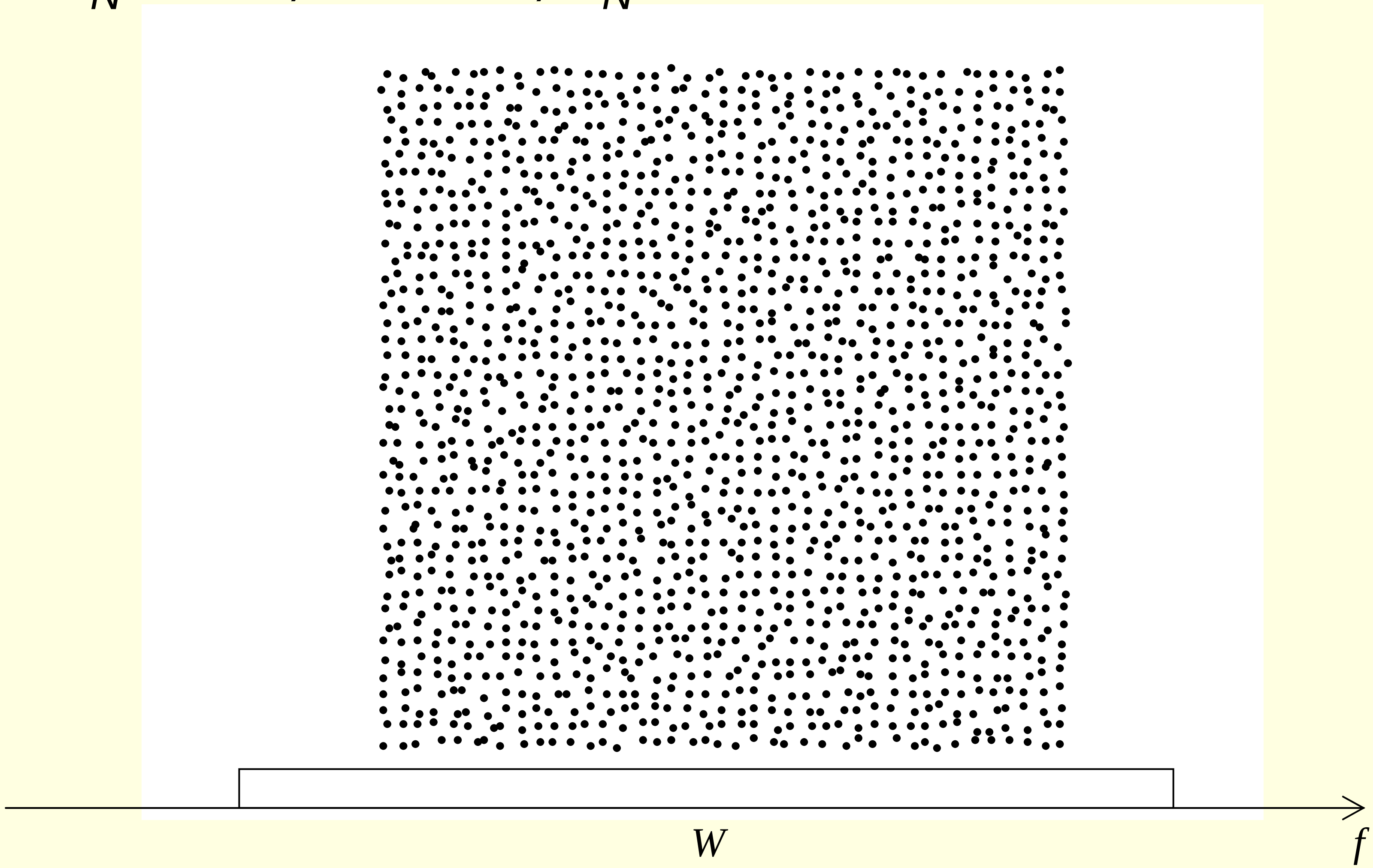


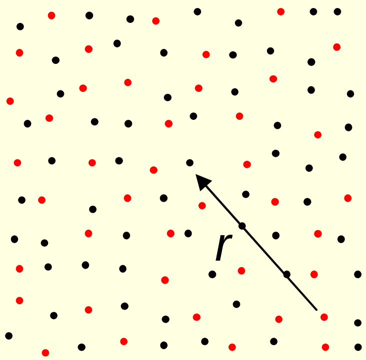
Objective

- For the limiting network as $d_N \rightarrow \infty$, $N \rightarrow \infty$, $d_N/N \rightarrow \alpha$, we derive
  - Transport capacity (bit-meters/symbol period/node)
  - Spectral efficiency (bit-meters/Hz/second/m^2)

Received Signal in a node

- Chip matched filter output in a receiving node

$$\mathbf{y} = b\sqrt{P(r)}\mathbf{s} + \sum_{\mathbf{x}\in B_N(t)} b_{\mathbf{x}}\sqrt{P(\|\mathbf{x}\|)}\mathbf{s}_{\mathbf{x}} + \mathbf{n}$$



  - *r* is link distance
  - *b*, *P*(*r*), and **s** are for desired sending node
  - $b_{\mathbf{x}}$, $P(\|\mathbf{x}\|)$, and $\mathbf{s}_{\mathbf{x}}$ are for interference nodes
  - $\mathbf{n} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$

MF Output

- MF outputs an estimate of *b*

$$y = \mathbf{s}^T \mathbf{y}$$

$$= \sqrt{P(r)}b + \sum_{\mathbf{x} \in B_N(t)} b_{\mathbf{x}} \sqrt{P(\|\mathbf{x}\|)} \mathbf{s}^T \mathbf{s}_{\mathbf{x}} + \mathbf{s}^T \mathbf{n}$$

$$= \sqrt{P(r)}b + I$$

- Unit-power SIR

$$\eta_N \equiv \frac{1}{E(I^2)}$$

Asymptotics

- Theorem: Interference *I* is asymptotically independent Gaussian, and unit-power SIR η_N converges a.s. to

$$\eta = \frac{1}{\sigma^2 + \overline{P}(\infty)}$$

- where total interference power to a node is finite

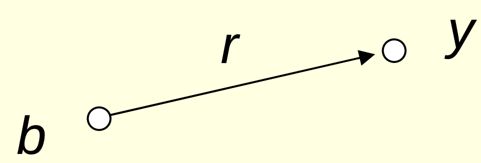
$$\overline{P}(\infty) = \frac{2\pi r_0^2 \alpha \rho P_0}{(\beta - 2)(\beta - 1)} \qquad \text{(watts/Hz/second)}$$

- Include all interference of the network
- Limit network is capable of information transportation

Limit Link Channel

- From sending *b* to MF output, there is a link channel, which is memoryless Gaussian

$$y = \sqrt{\eta P(r)}b + z$$



$z \sim N(0,1)$, i.i.d.

- SIR = $\eta P(r)$ depends only on link distance
- Same result can be obtained if a decorrelator or MMES receiver is employed

Link Channel Capacity

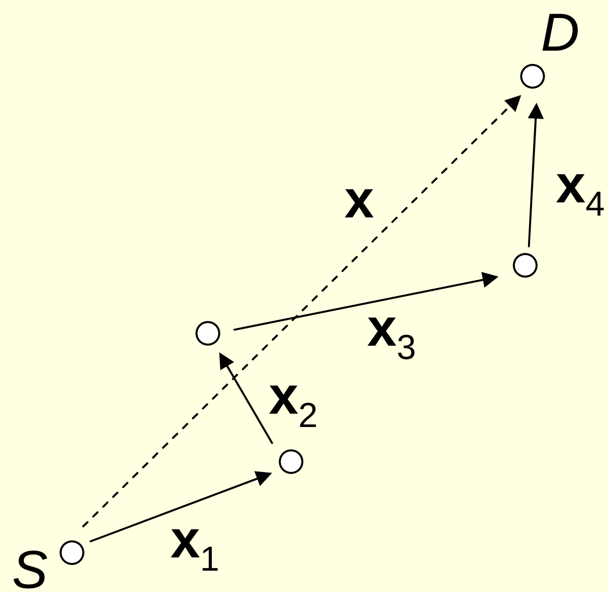
- For a link of distance *r*, the link capacity is

$$C(r) = \frac{1}{2}\log_2(1+\eta P(r))$$

(bits/symbol period)

Packet delivery

- A packet is delivered from source node to destination node via a multihop route
  $\varphi(\mathbf{x}) = \{\mathbf{x}_i, i = 1, \ldots, h(\mathbf{x}), \mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_{h(\mathbf{x})} = \mathbf{x}\}$
- A packet is coded with achievable rate
- The code rate of a packet to be delivered via route $\varphi(\mathbf{x})$ must be not greater than the minimum link capacity on the route

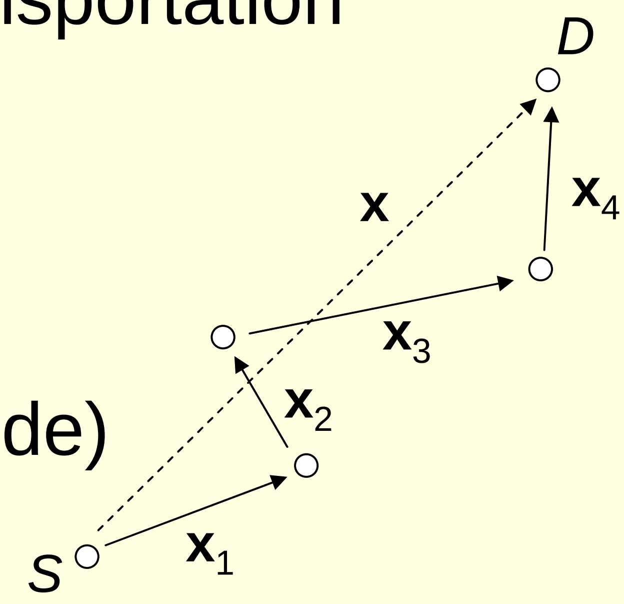


Route Transport Capacity

- Via route $\varphi(\mathbf{x})$, $\min_{1 \le i \le h(\mathbf{x})} C(\|\mathbf{x}_i\|)$ bits per symbol period are transported by a distance of $\|\mathbf{x}\|$ meters
- $h(\mathbf{x})$ nodes participate in transportation
- Route transport capacity is

$$\Gamma_{\varphi(\mathbf{x})} = \frac{\|\mathbf{x}\| \min_{1 \le i \le h(\mathbf{x})} C(\|\mathbf{x}_i\|)}{h(\mathbf{x})}$$

(bit-meters/symbol period/node)



Routing Protocol

- A global routing protocol schedules routes of all packets
- Consider achievable routing protocols that schedule routes without traffic conflict
- Let distribution of S-D vector **x** be $F(\mathbf{x})$
- For the same S-D vector **x**, different routes $\varphi(\mathbf{x})$ may be scheduled
- Under routing protocol u, let route $\varphi(\mathbf{x})$ for S-D vector **x** have distribution $V_u[\varphi(\mathbf{x})]$

Transport Throughput

- Transport throughput achieved under routing protocol *u*

$$\Gamma(u) = E_u\left(\rho\Gamma_{\varphi(\mathbf{r})}\right)$$
$$= \rho \int_{\Re^2} \int_{\varphi(\mathbf{x})\in\Omega_u(\mathbf{x})} \frac{\|\mathbf{x}\| \min_{1\le i\le h(\mathbf{x})} C(\mathbf{x})}{h(\mathbf{x})} dV_u(\varphi(\mathbf{x}))dF(\mathbf{x})$$

(bit-meters/symbol period/node)

- $F(\mathbf{x})$ – distribution of S-D vector **x**
- $V_u[\varphi(\mathbf{x})]$ – route distribution

Transport Capacity

- Each achievable routing protocol attains a transport throughput
- Transport capacity is defined as

$$\Gamma = \sup_{u \in \Psi} \Gamma(u)$$

  - Ψ – collection of all achievable routing protocols

Spectral Efficiency

- Given transport capacity Γ, spectral efficiency is

$$\Pi = \alpha\Gamma$$

(bit-meters/Hz/second/m^2)

Main Result

- Theorem: Transport capacity equals

$$\Gamma^* = \rho \int_0^\infty r \max_{h(r)\geq 1} \frac{C(r/h(r))}{h(r)} dF(r)$$

  - *r* – S-D distance; *F*(*r*) – distribution of *r*

- Spectral efficiency equals

$$\Pi^* = \alpha\rho \int_0^\infty r \max_{h(r)\geq 1} \frac{C(r/h(r))}{h(r)} dF(r)$$

Outline of Proof

- Step 1: Show that Γ^* is an upper bound
- Step 2: Show that Γ^* is the lowest upper bound
  - Need to find an achievable routing protocol to attain $\Gamma^* - \varepsilon$ for any $\varepsilon > 0$

Scaling Law

- If $\alpha \to \infty$ (or N fixed but $d_N \to \infty$), then

$$\Gamma = O(1/\alpha)$$

$$\Pi = O(1)$$

  - Transport capacity goes to zero at rate $1/\alpha$ - "scaling law" behavior
  - Spectral efficiency converges to a constant
- This scaling law is due to that radio bandwidth does not increases as fast as node density increases
  - – different from that of Gupta-Kumar model

Scaling Law

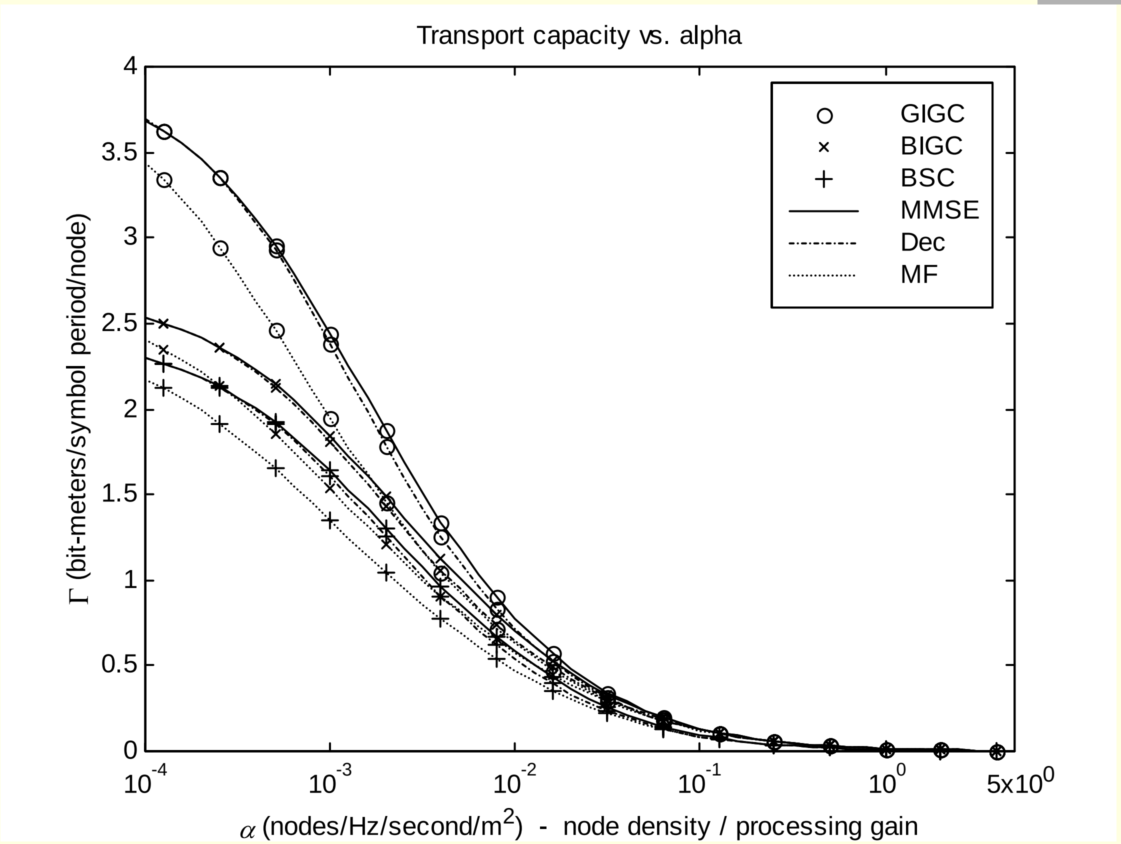
- The “scaling law” can be overcome, provided spreading gain N (or bandwidth) increases at the same rate as node density d_N increases

$$\Gamma = \text{constant} > 0$$

$$\Pi = \text{constant} > 0$$

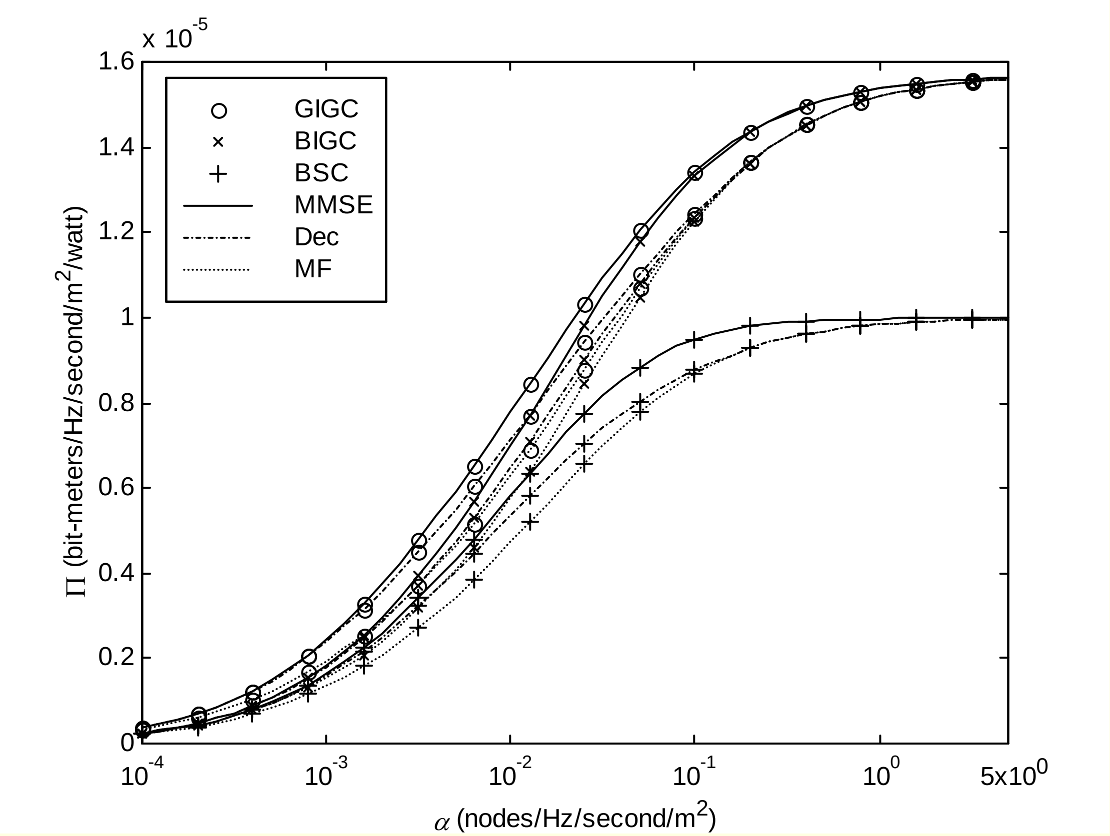
- A large wireless CDMA ad hoc network is capable of information transportation!

Transport Capacity vs. Traffic Intensity



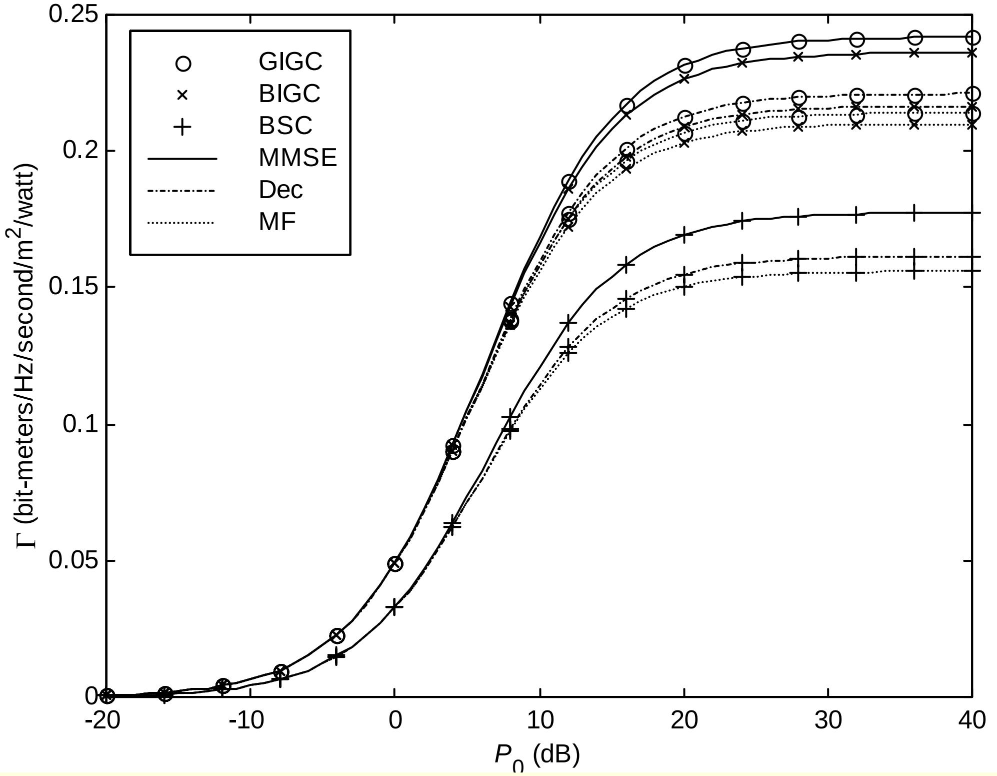
- Transport capacity monotonically decreases with α

Spectral Efficiency vs. Traffic Intensity



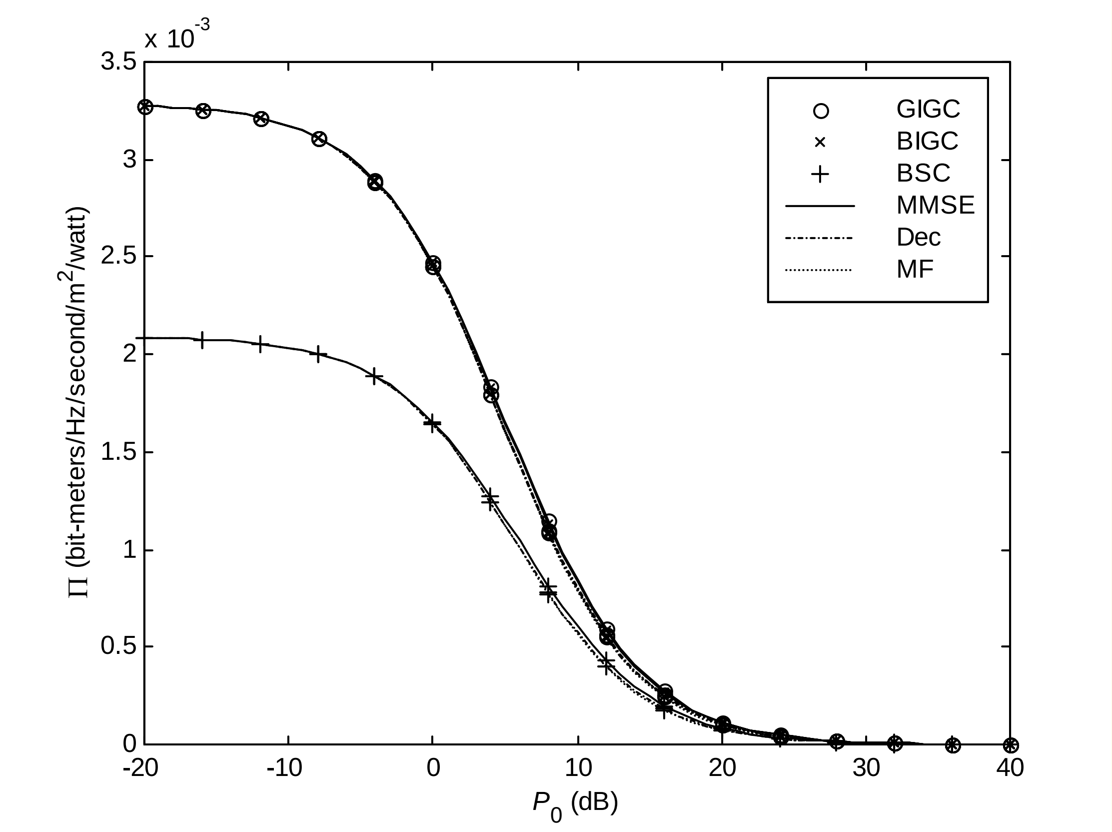
- Π monotonically increases with α

Transport Capacity vs. Transmission Power



- Transport capacity monotonically increases with P_0

Spectral Power Efficiency vs. Transmission Power



- Π monotonically decreases with P_0

Sensor Networks:
Sensor Density vs. Transmission Power

- Sensor network is low powered, $P_0 \to 0$
- Question: with given total power per square meter $\alpha\rho P_0 = \omega$,
  - should we increase node density and decrease node transmission power?
  - or converse?

$$\lim_{P_0 \to 0, \alpha\rho P_0 = \omega} \Pi = c\alpha\rho\eta_\omega \int_0^\infty \frac{r}{\min\limits_{h(r)\in Z^+} h(r)[r/(r_0 h(r)) + 1]^\beta} dR(r)$$

- Answer:
  - we should increase node density and decrease node transmission power in terms of increase of spectral power efficiency

Conclusions

- If radio bandwidth increases slower than node density increases, transport capacity decreases to zero – “scaling law”
  - The scaling law is essentially different from that of Gupta-Kumar model
  - The scaling law can be overcome, provided radio bandwidth increases as fast as node density increases
- A large wireless CDMA ad hoc network is capable of information transportation