Max SNR Array Processing for Space-Time Coded Systems

Hsuan-Jung Su
Department of Electrical Engineering
National Taiwan University
Outline

• Multi-Antenna Systems
• BLAST vs Space-Time Coding
• Rate-Performance Tradeoff: Grouped Space-Time Transmission
• Decoding of GST: Zero-Forcing vs Max SNR
• Performance
• Iterative Processing
• Conclusion
Multi-Antenna Systems

Array Processing, Coding

Array Processing
Channel Capacity

- MIMO Channel Capacity \((n \text{ Tx}, m \text{ Rx})\)
  \[
  C = \log \det \left( I_m + \left( \frac{\rho}{n} \right) \cdot HH^H \right)
  \]
  
  \(H\) : Channel Gain Matrix
  \(\rho\) : SNR

- Channel capacity increases linearly with \(\min(n,m)\) when the parallel channels are uncorrelated with one another.

- In terms of practical use, the increased capacity amounts to enhanced power efficiency and/or spectral efficiency.
Increased Capacity = ?

- **Spectral Efficiency (Multiplexing Gain):**
  Transmit as high data rate as possible but sacrifice power efficiency (e.g., BLAST).

- **Power Efficiency (Diversity Gain):**
  Tx/Rx array processing, Tx coding and modulation design to achieve high spatial diversity (e.g., space-time coding). Each information bit spreads over many Tx antennas. Spectral efficiency is low.
BLAST Transmission

Encoder

Demux

Y

Y

Y
BLAST Reception

SNR Ordering

Interference Suppression

Interference Cancellation

Mux

Decoder
Space-Time Coding

Source

S-T

ENC

R: 1/n

\[ \begin{array}{ccc}
  c_1^1 & c_2^1 & \cdots & c_1^1 \\
  c_1^2 & c_2^2 & \cdots & c_1^2 \\
  \vdots & \vdots & \ddots & \vdots \\
  c_1^n & c_2^n & \cdots & c_1^n \\
\end{array} \]
Space-Time Decoding

\[ P_e \leq a \cdot SNR^{-nm} \]
Rate-Performance Tradeoff:
Grouped Space-Time Transmission

For downlink in a cellular system, each group could be destined for one mobile.
Decoding of GST

Sequential Decoding: Each stage consists of group interference suppression, decoding, and cancellation of the decoded group.
GST Signal Model

\[ r_t = H c_t + n_t \]

\[ r_t = \left[ r_t^1, r_t^2, \Lambda, r_t^m \right]^T \quad \text{Received Signal} \]

\[ c_t = \left[ c_t^1, c_t^2, \Lambda, c_t^n \right]^T \quad \text{Simultaneously Transmitted Symbols} \]

\[ n_t = \left[ n_t^1, n_t^2, \Lambda, n_t^m \right]^T \quad \text{AWGN} \]

\[ H = \begin{bmatrix}
    h_{1,1} & h_{2,1} & \Lambda & h_{n,1} \\
    h_{1,2} & h_{2,2} & \Lambda & h_{n,2} \\
    M & M & O & M \\
    h_{1,m} & h_{2,m} & \Lambda & h_{n,m}
\end{bmatrix} \quad \text{Channel Gain Matrix} \]
Group Interference Suppression I
Zero-Forcing

When decoding group 1 (with $n_1$ transmit antennas), use the null space of the interference space as the array processor to remove completely signals from the other groups.

\[
\Lambda(C_1) = \begin{bmatrix}
    h_{n_1+1,1} & h_{n_1+2,1} & \Lambda & h_{n_1,1} \\
    h_{n_1+1,2} & h_{n_1+2,2} & \Lambda & h_{n_1,2} \\
    M & M & O & M \\
    h_{n_1+1,m} & h_{n_1+2,m} & \Lambda & h_{n_1,m}
\end{bmatrix}
\]

as the array processor to remove completely signals from the other groups.

- Requirement: $m \geq n - n_1 + 1$
- Diversity: $n_1(m - n + n_1)$
Group Interference Suppression II

Max SNR

Let $R_s$ and $R_n$ be the signal and interference covariance matrices, respectively, defined by

$$R_s = H(C_1)E[c_t^1(c_t^1)^H]H^H(C_1), \quad R_n = \Lambda(C_1)E[c_t^0(c_t^0)^H]\Lambda^H(C_1) + N_0 I$$

The linear maximum SNR array processor for this group consists of a set of $k$ linearly independent eigenvectors corresponding to the non-zero eigenvalues, counting multiplicities, of the generalized eigenvalue problem

$$R_s w = \lambda R_n w$$

where $k \leq \min(m, n_1)$ is the rank of $R_s$. The filtering outputs of these eigenvectors $\{w_i^H r_t\}_{i=1}^k$ are uncorrelated with one another.
**Zero-Forcing vs Max SNR**

Assume perfect cancellation after decoding each group, when decoding group $i$:

<table>
<thead>
<tr>
<th></th>
<th>Zero-Forcing</th>
<th>Max SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Requirement</td>
<td>$m \geq n - \sum_{j=1}^{i} n_j + 1$</td>
<td>$m \geq 1$</td>
</tr>
<tr>
<td>Diversity</td>
<td>$n_i(m - n + \sum_{j=1}^{i} n_j)$</td>
<td>$\geq n_i(m - n + \sum_{j=1}^{i} n_j)$</td>
</tr>
<tr>
<td>Signal Energy</td>
<td>Partially Collected</td>
<td>Fully Collected</td>
</tr>
<tr>
<td>No. of Filters</td>
<td>$m - n + \sum_{j=1}^{i} n_j$</td>
<td>$\leq \min(m, n_i)$</td>
</tr>
</tbody>
</table>
Without Interfering Groups

2 Tx 2 Rx Antennas

FER

SNR per Rx Antenna (dB)

Optimal

PRC

MSNR
Power Allocation (Open Loop)

- Zero-forcing diversity increases linearly with decoding stage $\rightarrow$ geometrically decreasing power allocation.
- Max SNR can use a more slowly decreasing power allocation (e.g., arithmetic).

Spatial Interleaving

The diversity gain after group-based spatial interleaving is no less than that provided by the space-time coding.
4 Groups (3 Are Interfering)

8 Tx 8 Rx Antennas

SNR per Rx Antenna (dB)

FER

ZF Geo. Power
ZF Ari. Power
MSNR Geo. Power
MSNR Ari. Power
Iterative Max SNR GST Receiver
Performance Comparison

8 Tx 8 Rx Antennas

SNR per Rx Antenna (dB)

FER

- ZF Geo.
- MSNR Geo.
- MSNR Ari.
- MSNR Ari. Int.
- MSNR Eq. Int. Iter.
Conclusion

• Grouped space-time transmission achieves a tradeoff between transmission rate and performance.

• Max SNR array processing for GST outperforms ZF, and subsumes PRC. It requires a different power allocation than ZF to achieve a further performance gain.

• Max SNR array processing allows iterative processing. It can also be made adaptive and semi-blind for downlink applications where a mobile is aware of only its own group.