Max SNR Array Processing for Space-Time Coded Systems

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Outline

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- BLAST vs Space-Time Coding
- Rate-Performance Tradeoff: Grouped Space-Time Transmission
- Decoding of GST: Zero-Forcing vs Max SNR
- Performance
- Iterative Processing
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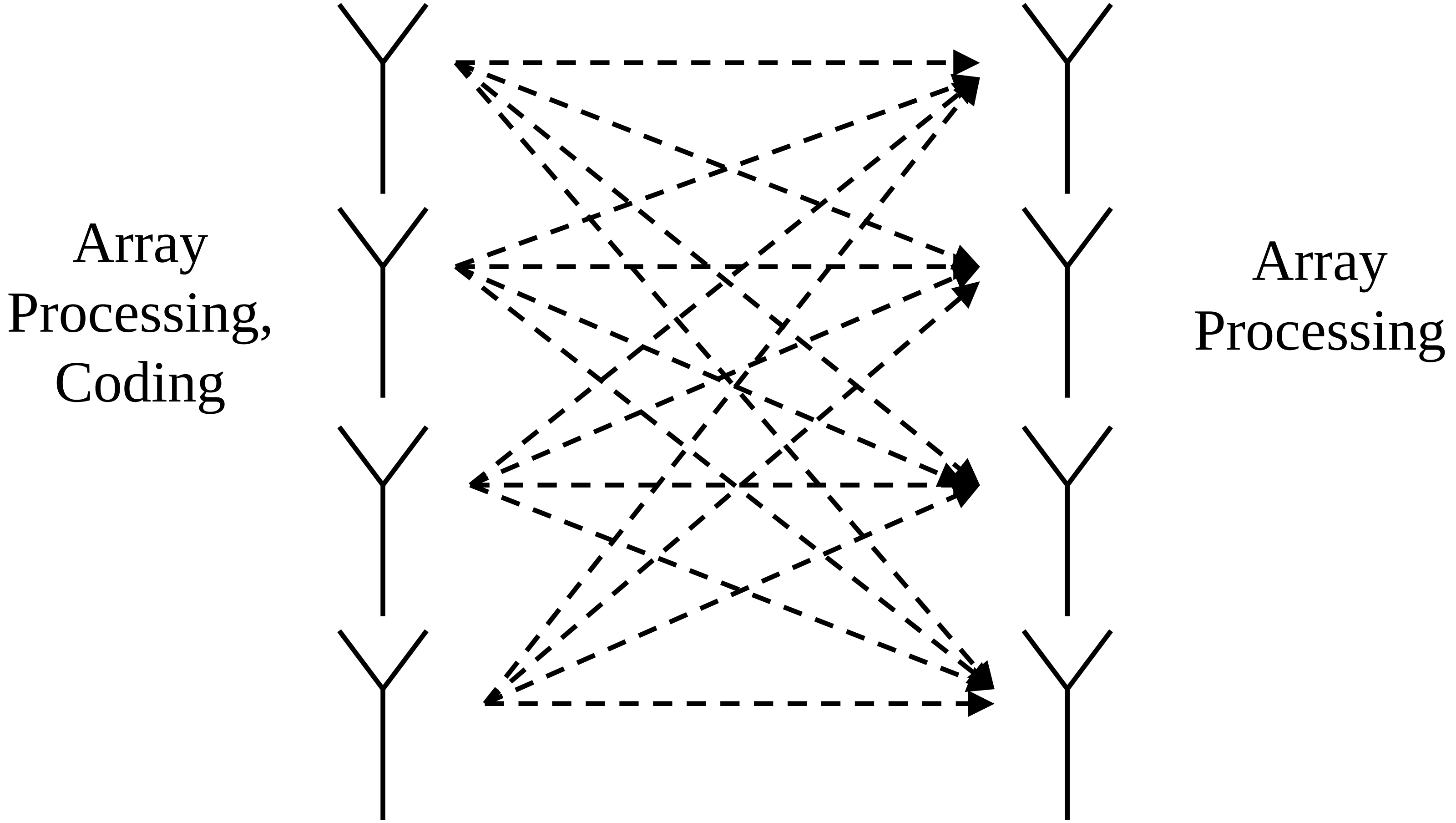
Multi-Antenna Systems



Channel Capacity

- MIMO Channel Capacity (*n* Tx, *m* Rx)

$$C = \log\det[I_m + (\rho / n) \cdot HH^H]$$

H : Channel Gain Matrix

ρ : SNR

- Channel capacity increases linearly with min(*n*,*m*) when the parallel channels are uncorrelated with one another.
- In terms of practical use, the increased capacity amounts to enhanced power efficiency and/or spectral efficiency.

Increased Capacity = ?

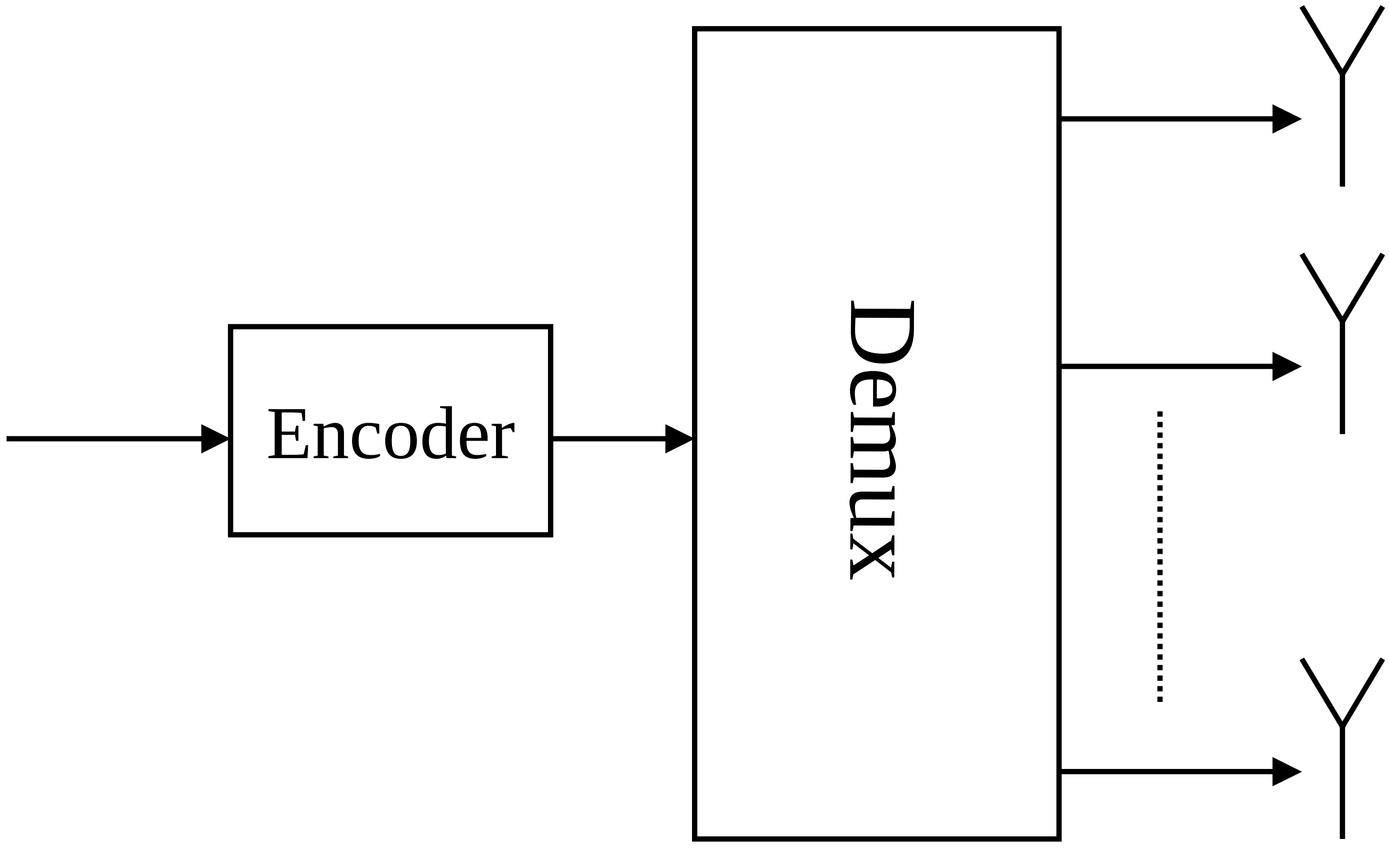
- Spectral Efficiency (Multiplexing Gain):

  Transmit as high data rate as possible but sacrifice power efficiency (e.g., BLAST).

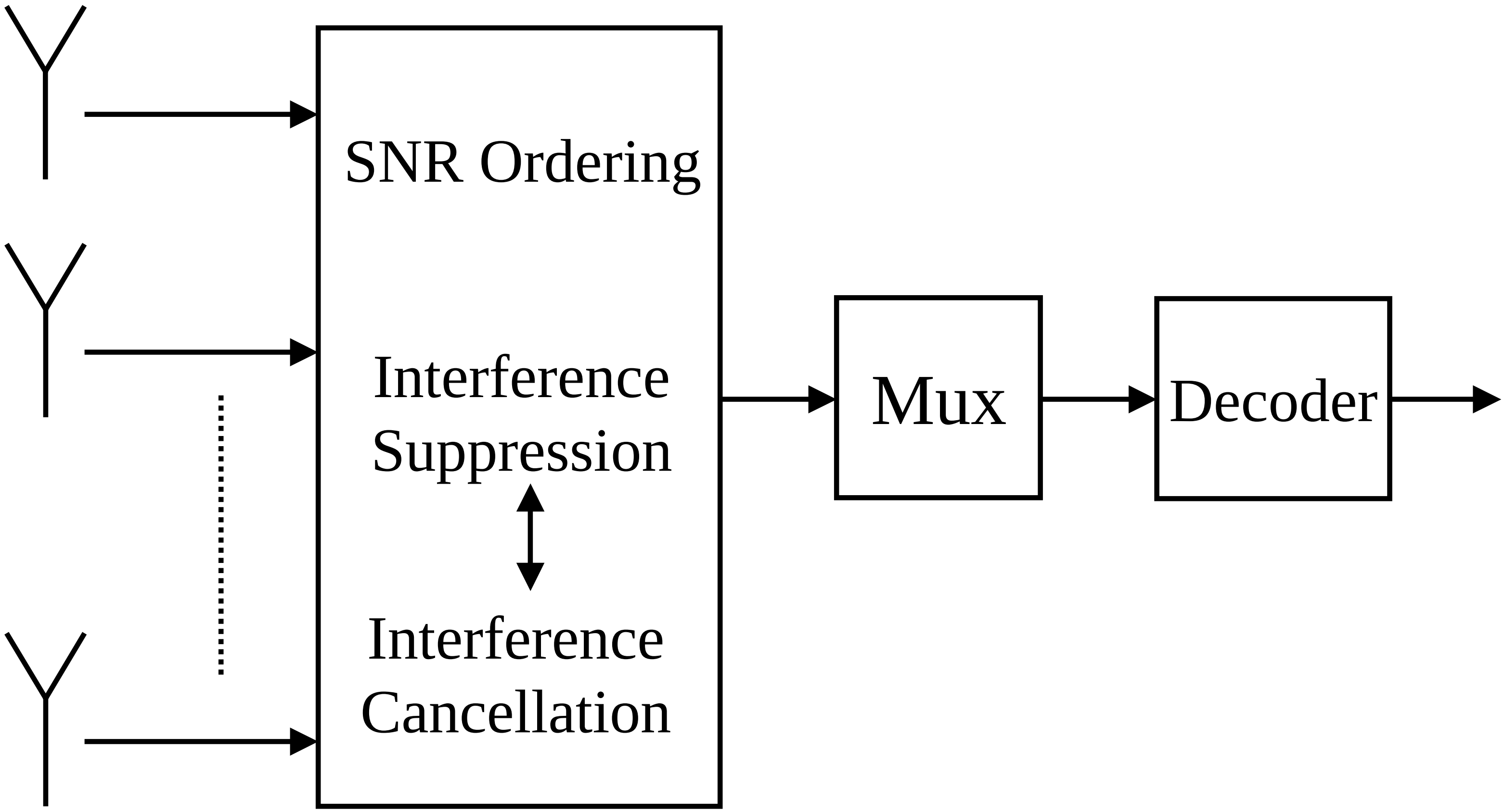
- Power Efficiency (Diversity Gain):

  Tx/Rx array processing, Tx coding and modulation design to achieve high spatial diversity (e.g., space-time coding). Each information bit spreads over many Tx antennas. Spectral efficiency is low.

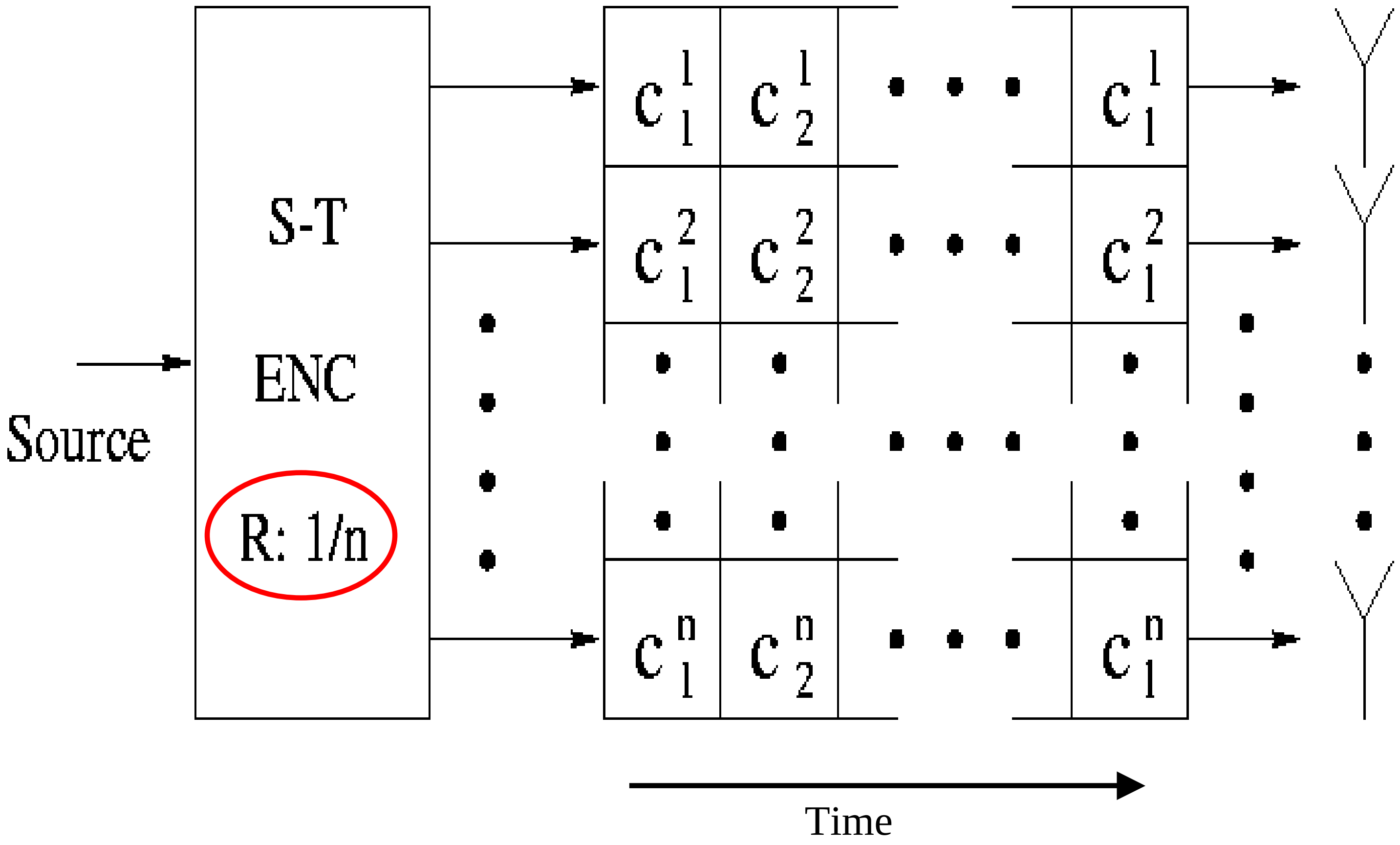
BLAST Transmission



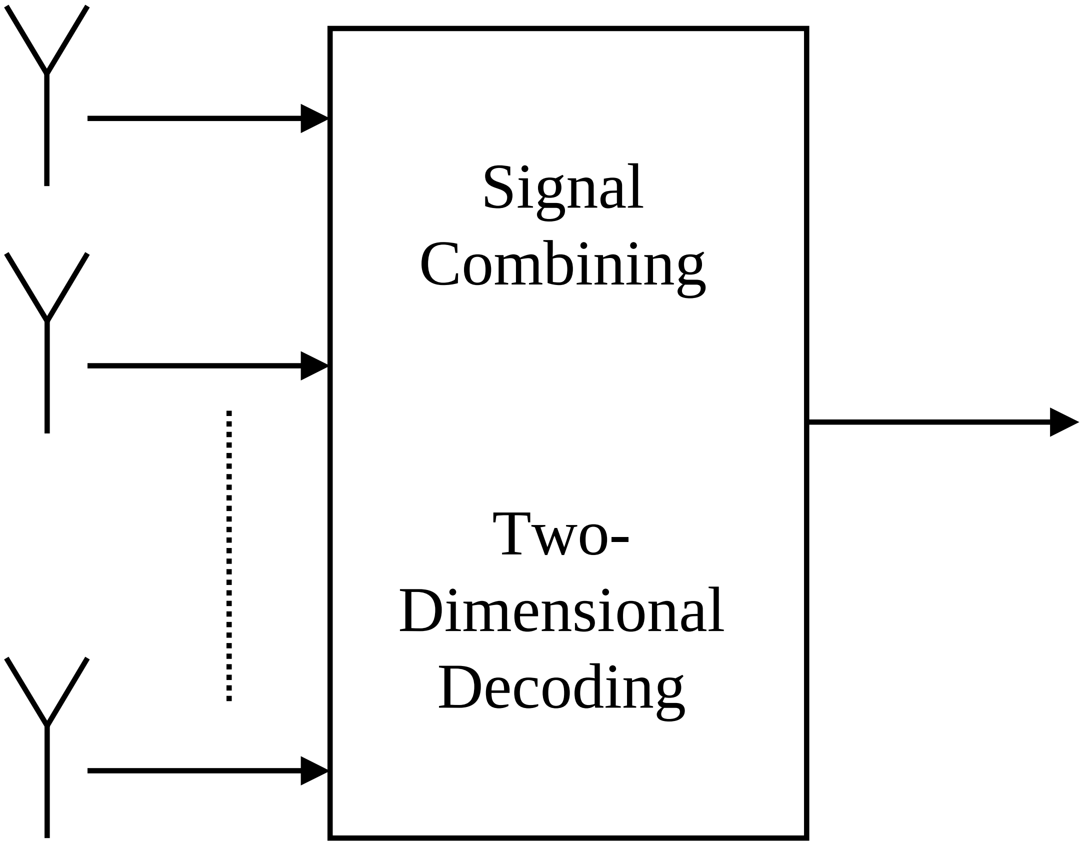
BLAST Reception



Space-Time Coding

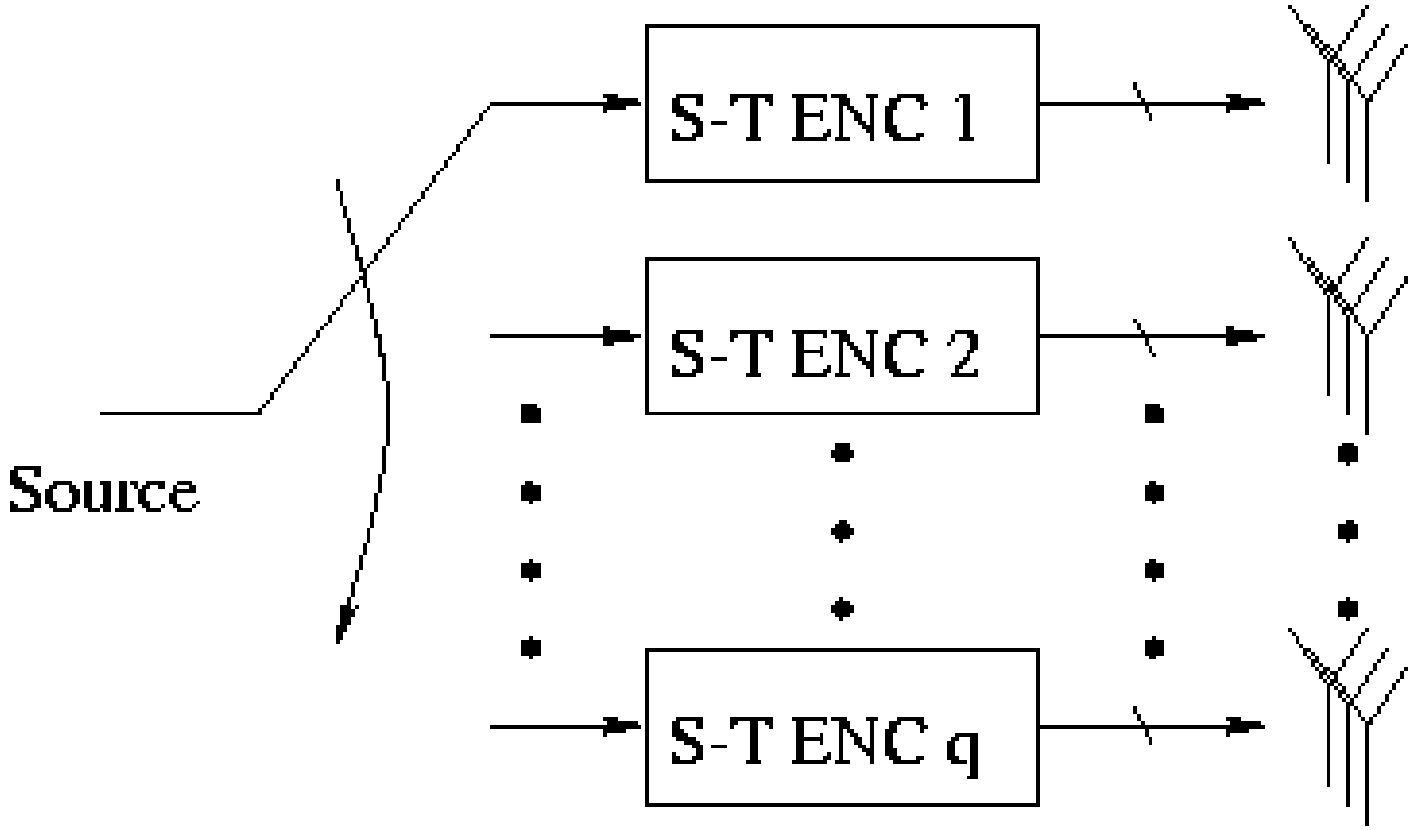


Space-Time Decoding



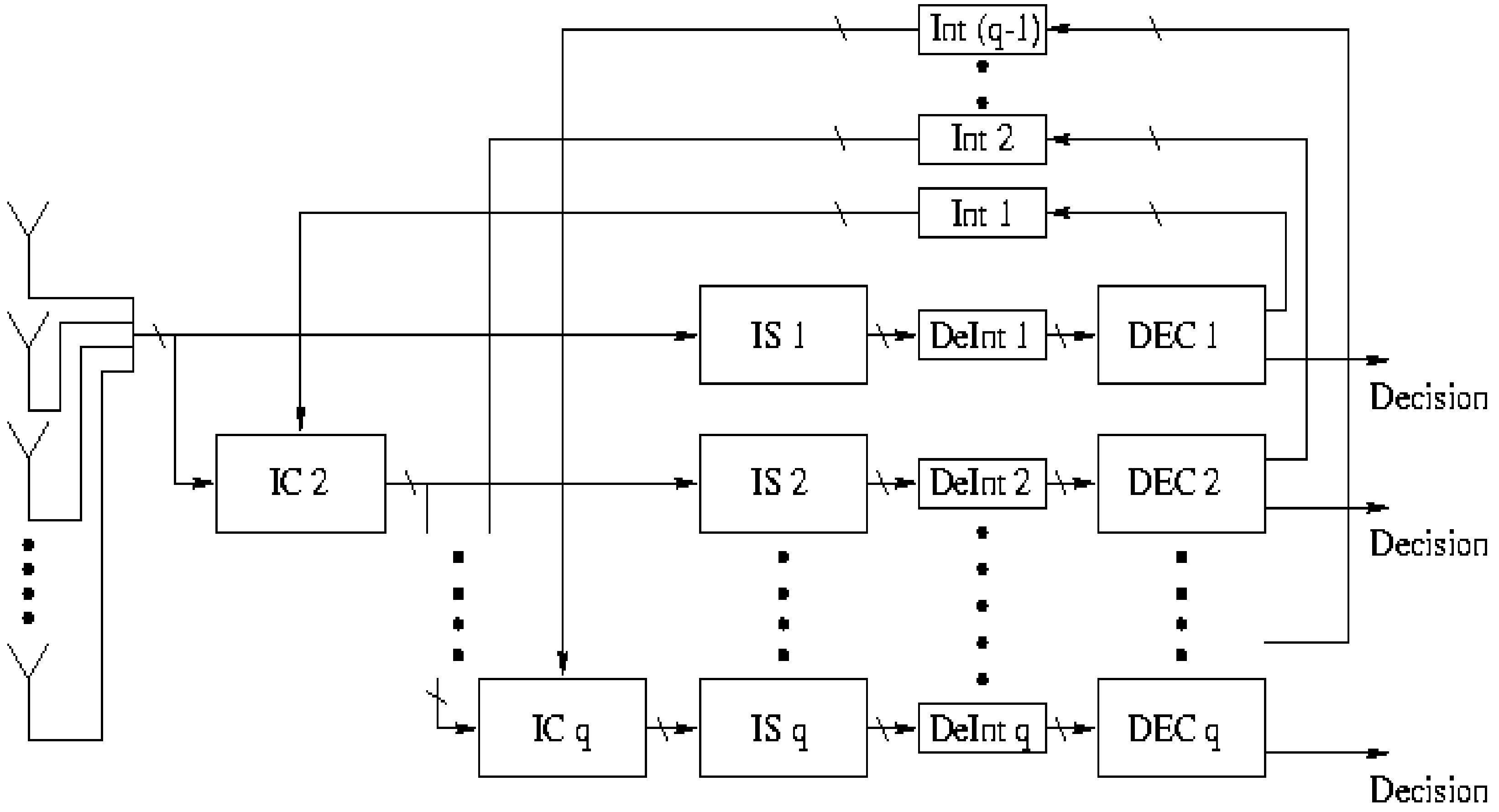
$$P_e \leq a \cdot SNR^{-nm}$$

Rate-Performance Tradeoff: Grouped Space-Time Transmission



For downlink in a cellular system, each group could be destined for one mobile.

Decoding of GST



Sequential Decoding: Each stage consists of group interference suppression, decoding, and cancellation of the decoded group.

GST Signal Model

$$r_t = Hc_t + n_t$$

$$r_t = \left[r_t^1, r_t^2, \cdots, r_t^m\right]^T$$ Received Signal

$$c_t = \left[c_t^1, c_t^2, \cdots, c_t^n\right]^T$$ Simultaneously Transmitted Symbols

$$n_t = \left[n_t^1, n_t^2, \cdots, n_t^m\right]^T$$ AWGN

$$H = \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{n,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,m} & h_{2,m} & \cdots & h_{n,m} \end{bmatrix}$$ Channel Gain Matrix

Group Interference Suppression I
Zero-Forcing

When decoding group 1 (with n_1 transmit antennas), use the null space of the interference space

$$\Lambda(C_1) = \begin{bmatrix} h_{n_1+1,1} & h_{n_1+2,1} & \cdots & h_{n,1} \\ h_{n_1+1,2} & h_{n_1+2,2} & \cdots & h_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_1+1,m} & h_{n_1+2,m} & \cdots & h_{n,m} \end{bmatrix}$$

as the array processor to remove completely signals from the other groups.

- Requirement: $m \geq n - n_1 + 1$
- Diversity: $\underline{n_1}\,\underline{(m - n + n_1)}$

Transmit Receive

Group Interference Suppression II
Max SNR

Let R_s and R_n be the signal and interference covariance matrices, respectively, defined by

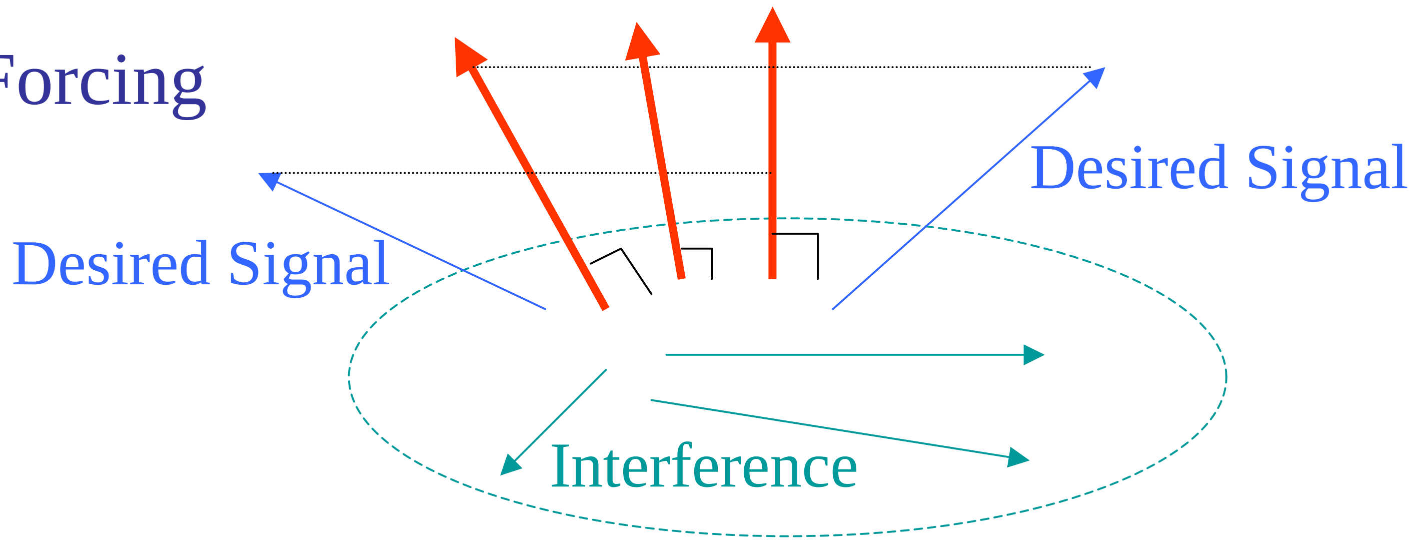
$$R_s = H(C_1)E[c_t^1(c_t^1)^H]H^H(C_1), \quad R_n = \Lambda(C_1)E[c_t^o(c_t^o)^H]\Lambda^H(C_1) + N_0 I$$

The linear maximum SNR array processor for this group consists of a set of k linearly independent eigenvectors corresponding to the non-zero eigenvalues, counting multiplicities, of the generalized eigenvalue problem

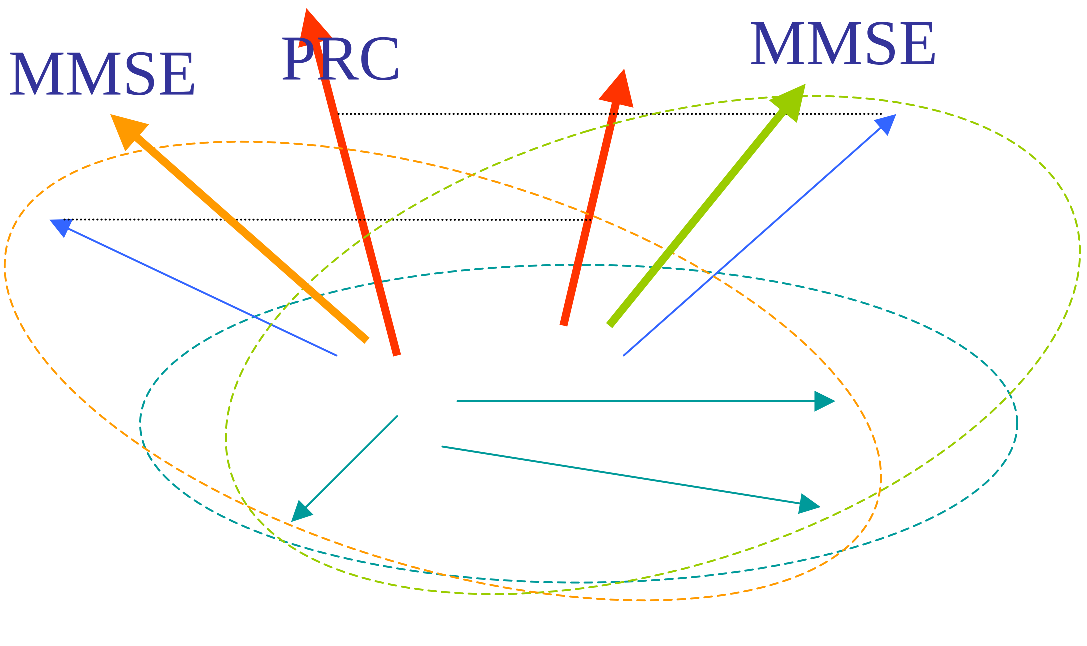
$$R_s w = \lambda R_n w$$

where $k \leq \min(m, n_1)$ is the rank of R_s. The filtering outputs of these eigenvectors $\{w_i^H r_t\}_{i=1}^k$ are uncorrelated with one another.

Zero-Forcing



Max SNR

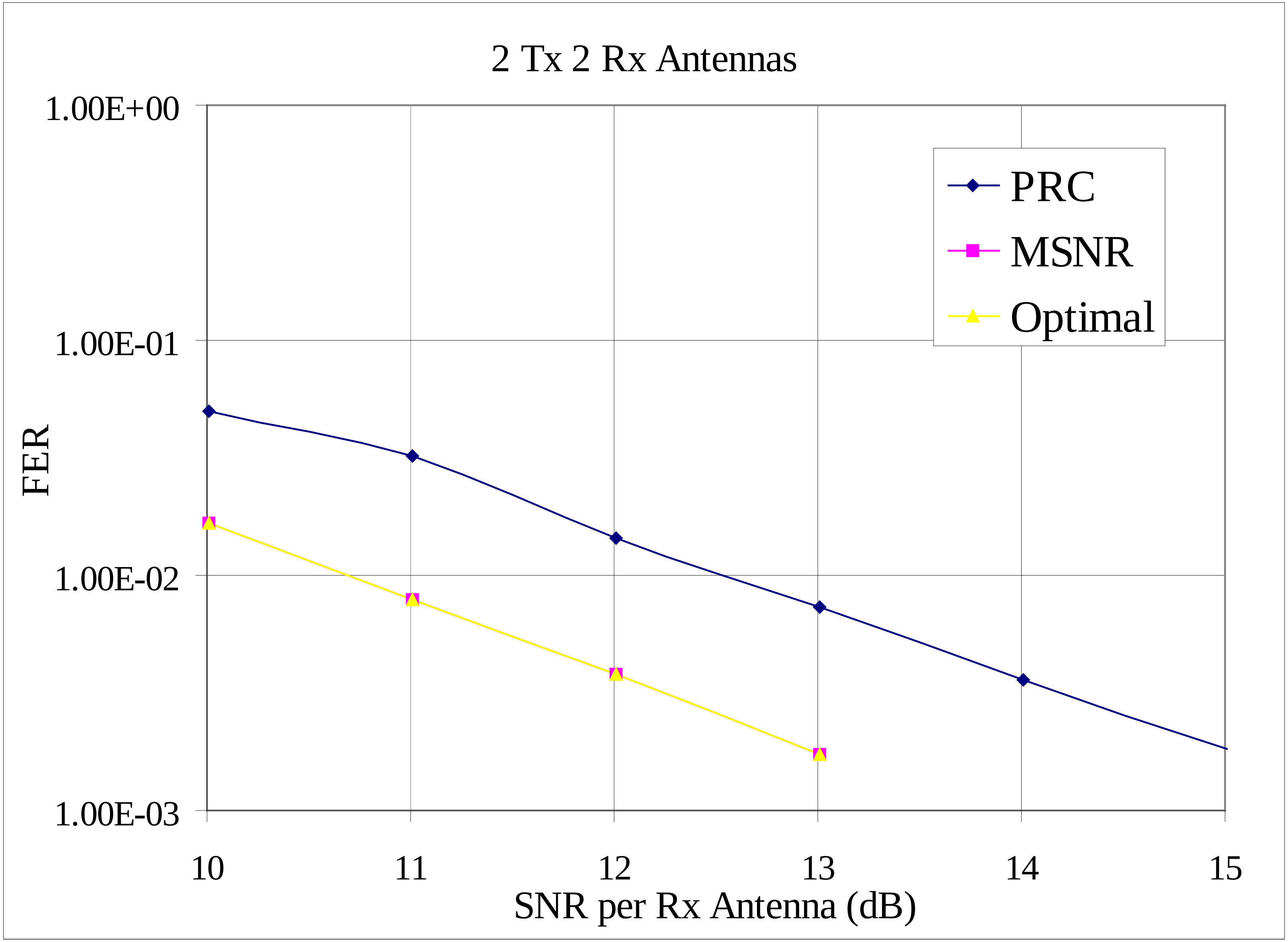


Zero-Forcing vs Max SNR

Assume perfect cancellation after decoding each group, when decoding group *i*:

	Zero-Forcing	Max SNR
Requirement	$m \geq n - \sum_{j=1}^{i} n_j + 1$	$m \geq 1$
Diversity	$n_i(m - n + \sum_{j=1}^{i} n_j)$	$\geq n_i(m - n + \sum_{j=1}^{i} n_j)$
Signal Energy	Partially Collected	Fully Collected
No. of Filters	$m - n + \sum_{j=1}^{i} n_j$	$\leq \min(m, n_i)$

Without Interfering Groups

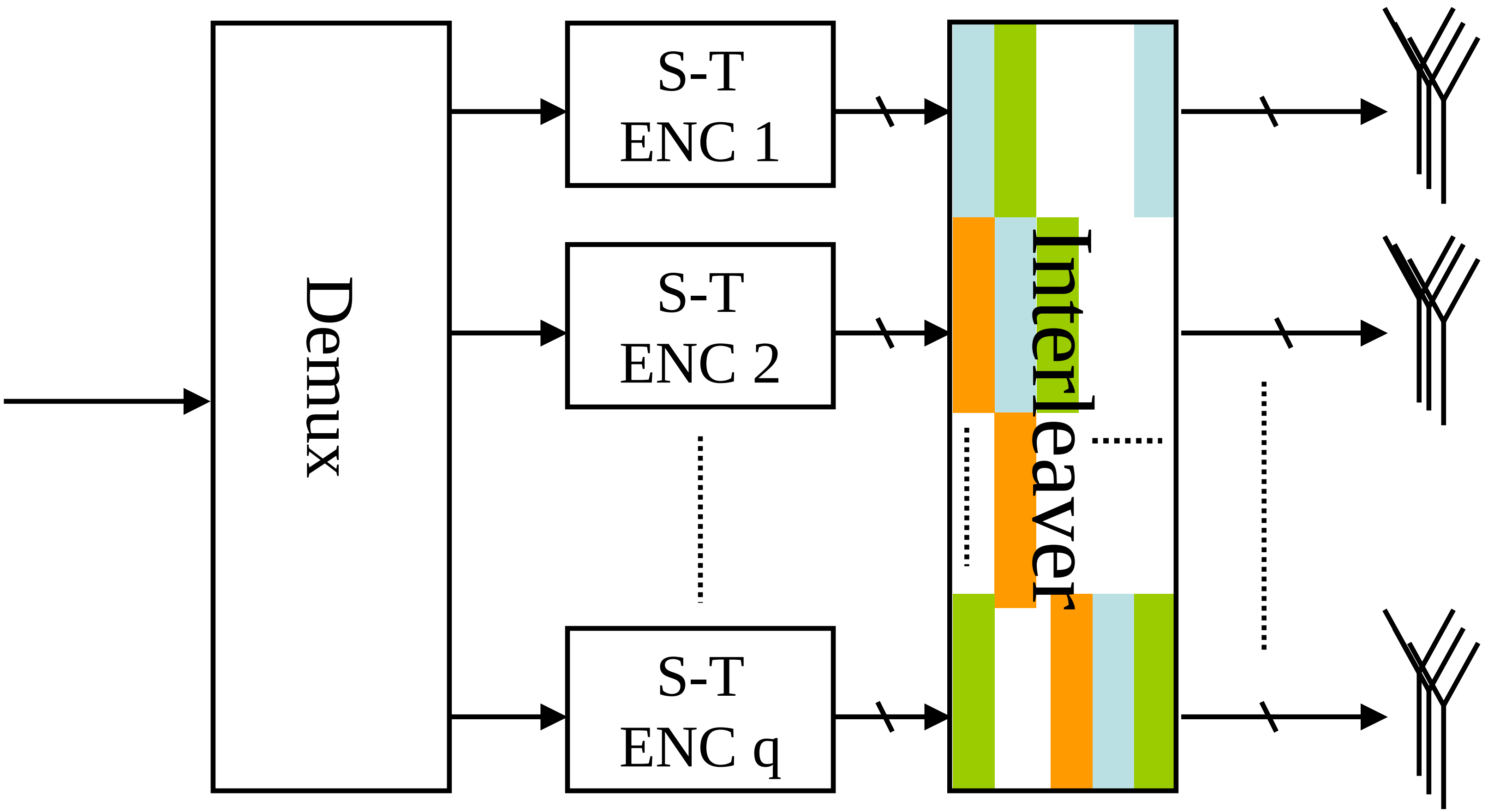


Power Allocation (Open Loop)

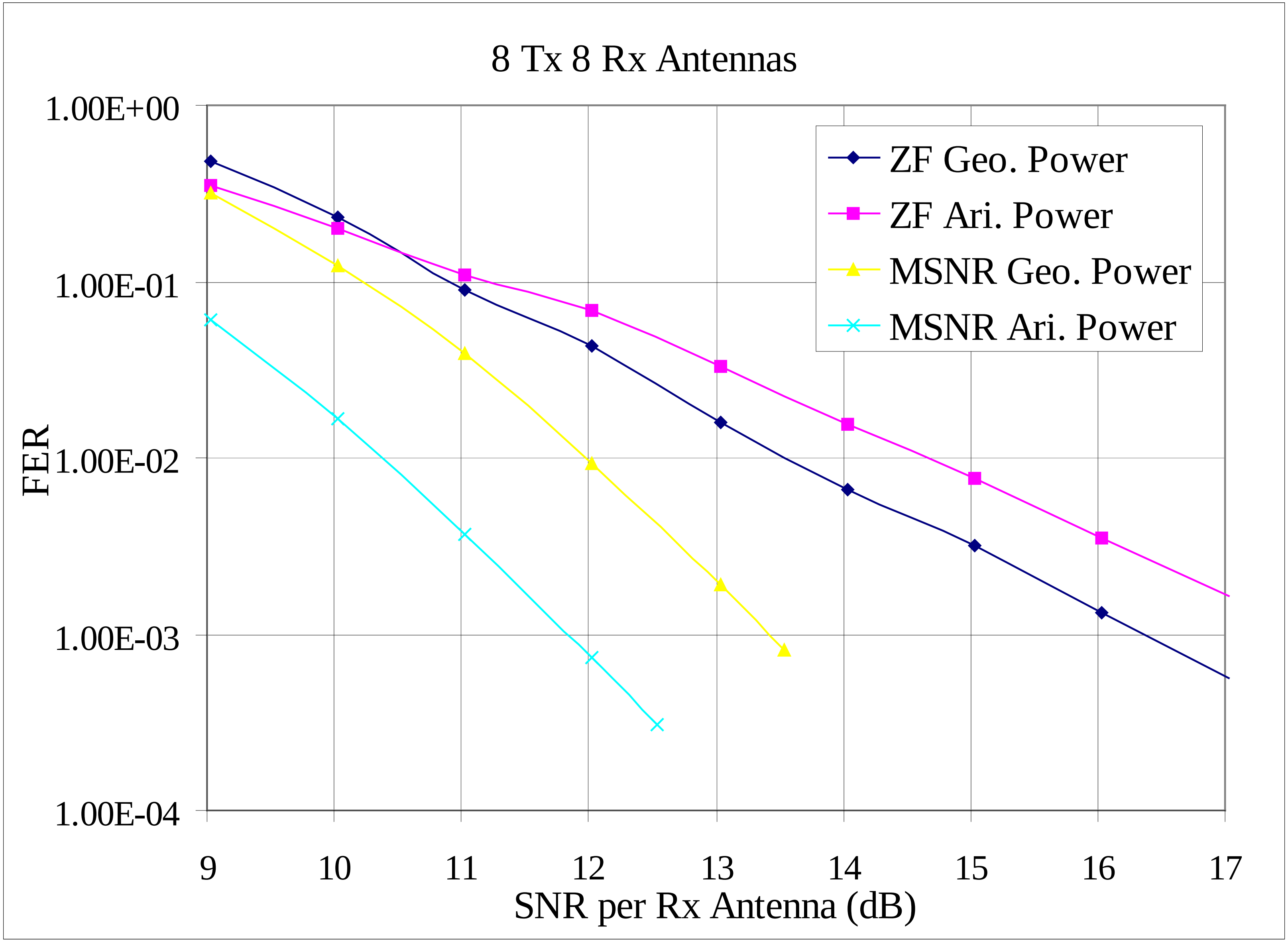
- Zero-forcing diversity increases linearly with decoding stage $\rightarrow$ geometrically decreasing power allocation.
- Max SNR can use a more slowly decreasing power allocation (e.g., arithmetic).

Spatial Interleaving

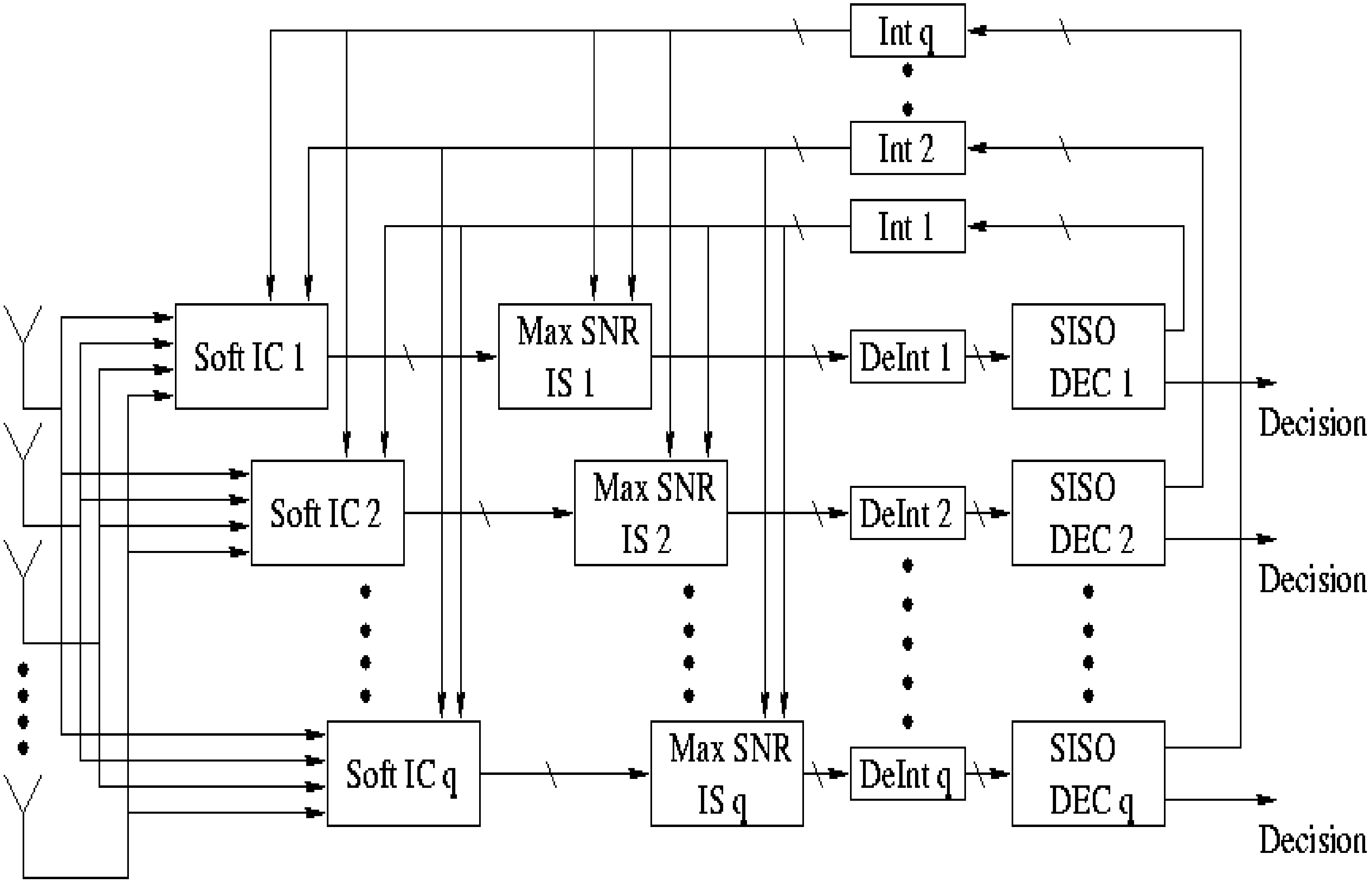
The diversity gain after *group-based* spatial interleaving is no less than that provided by the space-time coding.


Demux
S-T
ENC 1
S-T
ENC 2
S-T
ENC q
Interleaver

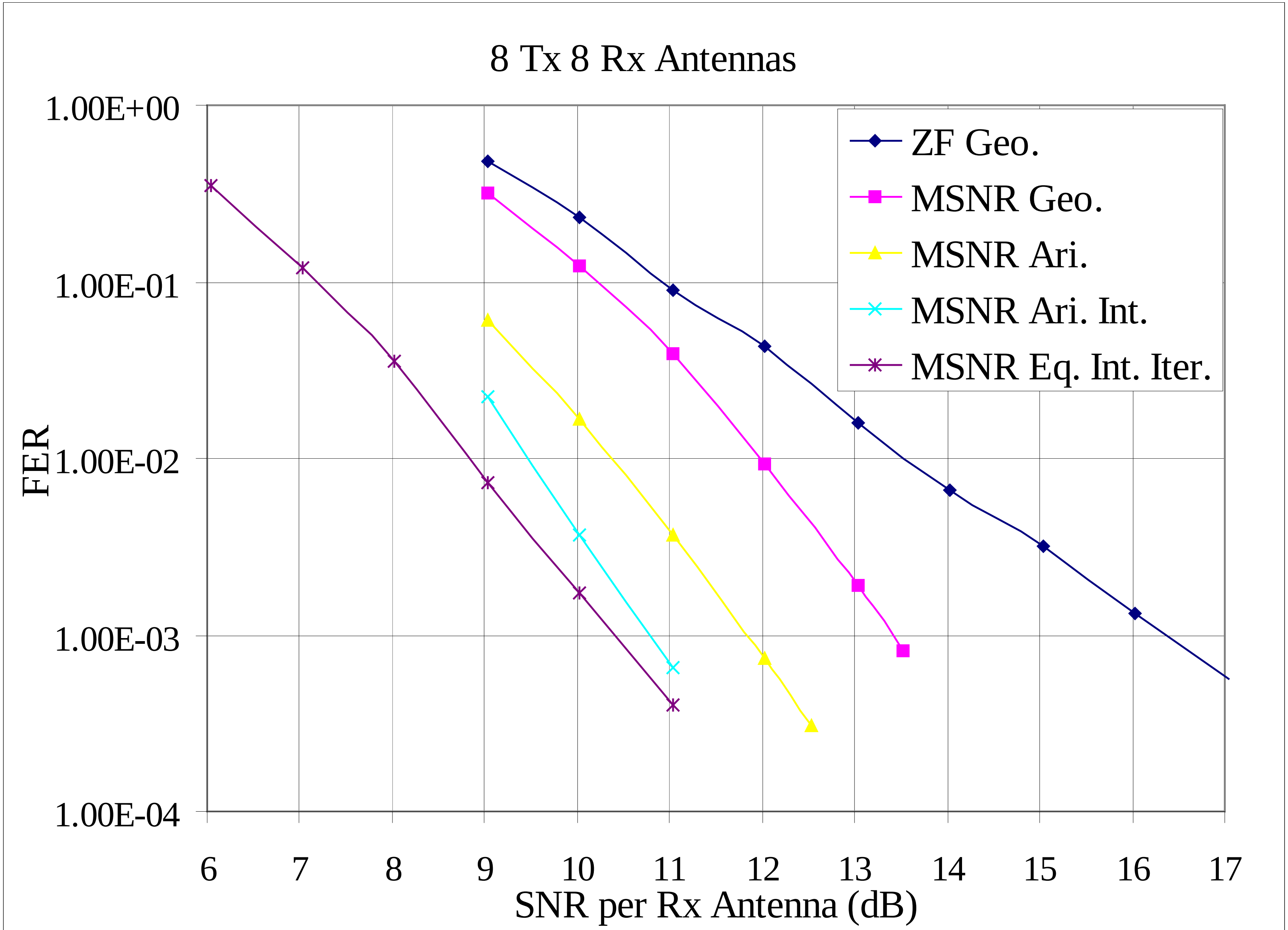
4 Groups (3 Are Interfering)



Iterative Max SNR GST Receiver



Performance Comparison



Conclusion

- Grouped space-time transmission achieves a tradeoff between transmission rate and performance.
- Max SNR array processing for GST outperforms ZF, and subsumes PRC. It requires a different power allocation than ZF to achieve a further performance gain.
- Max SNR array processing allows iterative processing. It can also be made adaptive and semi-blind for downlink applications where a mobile is aware of only its own group.