The Design of the Soft Decoder of the Interleaved Convolutional Code Used in IEEE 802.11a

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Outline

• Abstract.
• System model
• The design of the soft-decision decoding
• Simulation result
Abstract

• IEEE 802.11a incorporates high QAM to achieve a high data rate.
• A (2,1,7) convolutional code is used, and convolutional codes with higher rates are derived from it by employing “puncturing”.
• Soft-decision decoding instead of hard-decision decoding.
• The effects of block interleaving is also examined.
## IEEE 802.11a Spec.

<table>
<thead>
<tr>
<th>Data rate (Mbits/s)</th>
<th>Modulation</th>
<th>Coding rate (R)</th>
<th>Coded bits per OFDM symbol</th>
<th>Data bits per OFDM symbol</th>
<th>Block Interleaver</th>
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<tbody>
<tr>
<td>24</td>
<td>16-QAM</td>
<td>1 / 2</td>
<td>192</td>
<td>96</td>
<td>12 x 16</td>
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<tr>
<td>36</td>
<td>16-QAM</td>
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<td>12 x 16</td>
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<tr>
<td>48</td>
<td>64-QAM</td>
<td>2 / 3</td>
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<td>192</td>
<td>18 x 16</td>
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<td>54</td>
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<td>288</td>
<td>216</td>
<td>18 x 16</td>
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</table>
Data Rate 24Mbits/s

- C.C. : Convolutional Code
- V.D. : Viterbi Decoder
Data Rate 36Mbits/s

- C.C. : Convolutional Code
- V.D. : Viterbi Decoder
Data Rate 48Mbits/s

- C.C.: Convolutional Code
- V.D.: Viterbi Decoder

C.C. (2,1,7) → Puncturing (2,1,7) to (3,2,7) → Interleaving (18 x 16) → Modulation (64-QAM) → Channel

V.D. ← Depuncturing ← Deinterleaving ← Demodulation
Data Rate 54Mbits/s

- C.C. : Convolutional Code
- V.D. : Viterbi Decoder
(2,1,7) Convolutional Code

- (2,1,7) Convolutional Code,
  Constraint Length = 7
- Generators: 133, 171 in Octal
- Free Distance $d_{\text{free}} = 10$
Punctured (3,2,7) Convolutional Code

Source Data

Stolen Bit

Encoded Data

Bit Stolen Data

Inserted Dummy Bit

Bit Inserted Data

Decoded Data
Punctured (4,3,7) Convolutional Code

Source Data

Stolen Bit

Encoded Data

Bit Stolen Data

Inserted Dummy Bit

Bit Inserted Data

Decoded Data
Interleaving in IEEE 802.11a

- A two-step interleaving is designed.
  1. Mapping adjacent coded bits onto non-adjacent coded bits.
  2. Swapping the coded bits alternately onto less significant bits (LSB) and more significant bits (MSB) of the QAM constellation.
12 by 16 Block Interleaver (16QAM)

<table>
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<th>C2</th>
<th>C19</th>
<th>C4</th>
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<th>C6</th>
<th>C23</th>
<th>C8</th>
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<td>C188</td>
<td>C173</td>
<td>C190</td>
<td>C175</td>
</tr>
</tbody>
</table>
18 by 16 Block Interleaver (64QAM)
Soft-decision decoding

• In IEEE 802.11a, the hard-decision decoding is used because of the employment of high QAM and interleaving.
• The soft-decision decoding usually performs better than the hard-decision decoding.
• In this study, a soft-decision decoder is proposed by determining the bit log-likelihood (soft matrix) of each coded bit of the convolutional code.
16-QAM Constellation

$$b_1b_0$$

00 01 11 10

-3\sqrt{E} -\sqrt{E} +\sqrt{E} +3\sqrt{E}
Optimum Decision Rule for 16QAM(AWGN)

$b_0$: The less significant bit

\[
\Lambda(r|b_0=0) = \log \left[ e^{\frac{-(r+3\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{-(r-3\sqrt{\frac{2E}{N_0}})^2}{2}} \right]
\]

\[
\Lambda(r|b_0=1) = \log \left[ e^{\frac{-(r+\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{-(r-\sqrt{\frac{2E}{N_0}})^2}{2}} \right]
\]
Optimum Decision Rule for 16QAM(AWGN)

\( b_1 : \text{The more significant bit} \)

\[
\Lambda(r | b_1 = 0) = \log \left[ e^{-\frac{(r+3\sqrt{\frac{2E}{N0}})^2}{2}} + e^{-\frac{(r+\sqrt{\frac{2E}{N0}})^2}{2}} \right]
\]

\[
\Lambda(r | b_1 = 1) = \log \left[ e^{-\frac{(r-3\sqrt{\frac{2E}{N0}})^2}{2}} + e^{-\frac{(r-\sqrt{\frac{2E}{N0}})^2}{2}} \right]
\]
Sub-optimum dual-max Decision Rule
for 16QAM

\[
\Lambda(r|b_0 = 0) \approx \max \left\{ 3 \sqrt{\frac{2E}{N_0} r - \frac{9E}{2N_0}}, \ -3 \sqrt{\frac{2E}{N_0} r - \frac{9E}{2N_0}} \right\}
\]

\[
\Lambda(r|b_0 = 1) \approx \max \left\{ \sqrt{\frac{2E}{N_0} r - \frac{E}{2N_0}}, \ - \sqrt{\frac{2E}{N_0} r - \frac{E}{2N_0}} \right\}
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\Lambda(r|b_1 = 0) \approx \max \left\{ -3 \sqrt{\frac{2E}{N_0} r - \frac{9E}{2N_0}}, \ - \sqrt{\frac{2E}{N_0} r - \frac{E}{2N_0}} \right\}
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\Lambda(r|b_1 = 1) \approx \max \left\{ 3 \sqrt{\frac{2E}{N_0} r - \frac{9E}{2N_0}}, \ \sqrt{\frac{2E}{N_0} r - \frac{E}{2N_0}} \right\}
\]
64-QAM Constellation

$I_0 I_1 I_2$

110 111 101 100 000 001 011 010

$-7\sqrt{E} -5\sqrt{E} -3\sqrt{E} -\sqrt{E} +\sqrt{E} +3\sqrt{E} +5\sqrt{E} +7\sqrt{E}$
Optimum Decision Rule for 64QAM(AWGN)

\[
\begin{align*}
\Lambda(r | I_0 = 0) &= \log(e^{-\frac{(r-\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+3\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} ) \\
\Lambda(r | I_0 = 1) &= \log(e^{-\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} ) \\
\Lambda(r | I_1 = 0) &= \log(e^{-\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} ) \\
\Lambda(r | I_1 = 1) &= \log(e^{-\frac{(r+5\sqrt{2E}{N_0})^2}{2}} + e^{\frac{(r+5\sqrt{2E}{N_0})^2}{2}} + e^{\frac{(r-\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} ) \\
\Lambda(r | I_2 = 0) &= \log(e^{-\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} ) \\
\Lambda(r | I_2 = 1) &= \log(e^{-\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+5\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} + e^{\frac{(r+7\sqrt{\frac{2E}{N_0}})^2}{2}} )
\end{align*}
\]
Sub-optimum Dual-max Decision Rule
for 64QAM(AWGN)

\[
\Lambda(r|I_0 = 0) \approx \max \left\{ \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0}, \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0}, \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0} \right\}
\]

\[
\Lambda(r|I_0 = 1) \approx \max \left\{ -r \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, -3r \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0}, -5r \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0}, -7r \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0} \right\}
\]

\[
\Lambda(r|I_1 = 0) \approx \max \left\{ -3r \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0}, -r \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, r \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, 3r \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0} \right\}
\]

\[
\Lambda(r|I_1 = 1) \approx \max \left\{ -7r \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0}, -5r \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0}, 5r \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0}, 7r \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0} \right\}
\]

\[
\Lambda(r|I_2 = 0) \approx \max \left\{ -7r \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0}, -r \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, r \sqrt{\frac{2E}{N_0}} - \frac{E}{N_0}, 7r \sqrt{\frac{2E}{N_0}} - 49 \frac{E}{N_0} \right\}
\]

\[
\Lambda(r|I_2 = 1) \approx \max \left\{ -5r \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0}, -3r \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0}, 3r \sqrt{\frac{2E}{N_0}} - 9 \frac{E}{N_0}, 5r \sqrt{\frac{2E}{N_0}} - 25 \frac{E}{N_0} \right\}
\]
Data rate 24Mbps: CC(2,1,7) + 16 QAM
Data rate 36Mbps : CC(4,3,7) + 16 QAM

![Error Performance in AWGN (4,3,7) 16QAM Graph](image)
Data rate 48Mbps: CC(3,2,7) + 64 QAM
Data rate 54Mbps: CC(4,3,7)+64 QAM
Effects of Interleaving

• Breaking the adjacent coded bits to reduce the risk of bursty errors.

• However, interleaving might hurt the error performance of the (2,1,7) convolutional code when the channel is AWGN. This is because no burty error is observed at the receiver even when the coded bits are not interleaved.

• This might be a special case when the channel is AWGN.

• The help of interleaving becomes significant when channel fading exists.
CC( 2,1,7 ) +16 QAM over AWGN
Rayleigh Fading Channel

Suboptimum decision rule in the Rayleigh fading channel

BER vs. Eb/No for different conditions.
Exponential Delay Power Profile

The multipath fading channel model proposed in IEEE 802.11a.
Multipath Fading Channel

Error performance in Multipath Fading Channel

- interleaving
- noninterleaving

BER vs. Eb/No graph
Conclusion

• The soft-decision decoding performs better than the hard-decision decoding designed in IEEE 802.11a.

• The sub-optimal dual-max decision rule that requires less complexity shows no significant difference in error performance over the optimal decision rule.

• The effects of interleaving has been examined under different channel environments.